**1.** In an arithmetic sequence, *u*1 = 2 and *u*3 = 8.

(a) Find *d*.

(2)

(b) Find *u*20.

(2)

(c) Find *S*20.

(2)

(Total 6 marks)

**2.** Let *f*(*x*)= 3 ln *x* and *g*(*x*) = ln 5*x*3.

(a) Express *g*(*x*) in the form *f*(*x*)+ ln *a*, where *a*  +.

(4)

(b) The graph of *g* is a transformation of the graph of *f*. Give a full geometric description of this transformation.

(3)

(Total 7 marks)

**3.** (a) Expand (2 + *x*)4 and simplify your result.

(3)

(b) Hence, find the term in *x*2 in (2 + *x*)4.

(3)

(Total 6 marks)

**4.** Let *f*(*x*) = log3, for *x* > 0.

(a) Show that *f*–1(*x*) = 32*x*.

(2)

(b) Write down the range of *f*–1.

(1)

 Let *g*(*x*) = log3 *x*, for *x* > 0.

(c) Find the value of (*f* –1 ° *g*)(2), giving your answer as an integer.

(4)

(Total 7 marks)

**5.** Solve log2*x* + log2(*x* – 2) = 3, for *x* > 2.

(Total 7 marks)

**6.** The first three terms of an infinite geometric sequence are 32, 16 and 8.

(a) Write down the value of *r*.

(1)

(b) Find *u*6.

(2)

(c) Find the sum to infinity of this sequence.

(2)

(Total 5 marks)

**7.** The fifth term in the expansion of the binomial (*a* + *b*)*n* is given by .

(a) Write down the value of *n*.

(1)

(b) Write down *a* and *b*, in terms of *p* and/or *q*.

(2)

(c) Write down an expression for the sixth term in the expansion.

(3)

(Total 6 marks)

**8.** (a) Find log2 32.

(1)

(b) Given that log2  can be written as *px* + *qy*, find the value of *p* and of *q*.

(4)

(Total 5 marks)

**9.** Let *f*(*x*) = *k* log2 *x*.

(a) Given that *f*–1(1) = 8, find the value of *k*.

(3)

(b) Find *f*–1.

(4)

(Total 7 marks)

**10.** Consider the arithmetic sequence 2, 5, 8, 11, ....

(a) Find *u*101.

(3)

(b) Find the value of *n* so that *un* = 152.

(3)

(Total 6 marks)

**11.** Consider the infinite geometric sequence 3, 3(0.9), 3(0.9)2, 3(0.9)3, … .

(a) Write down the 10th term of the sequence. Do not simplify your answer.

(1)

(b) Find the sum of the infinite sequence.

(4)

(Total 5 marks)

**12.** In an arithmetic sequence *u*21 = –37 and *u*4 = –3.

(a) Find

(i) the common difference;

(ii) the first term.

(4)

(b) Find *S*10.

(3)

(Total 7 marks)

**13.** Let *un* = 3 – 2*n*.

(a) Write down the value of *u*1, *u*2, and *u*3.

(3)

(b) Find .

(3)

(Total 6 marks)

**14.** Solve the following equations.

(a) log*x* 49 = 2

(3)

(b) log2 8 = *x*

(2)

(c) log25 *x* = 

(3)

(d) log2 *x* + log2(*x* – 7) = 3

(5)

(Total 13 marks)

**15.** Consider the infinite geometric sequence 25, 5, 1, 0.2, … .

(a) Find the common ratio.

(b) Find

(i) the 10th term;

(ii) an expression for the *n*th term.

(c) Find the sum of the infinite sequence.

(Total 6 marks)

**16.** Given that *p* = log*a* 5, *q* = log*a* 2, express the following in terms of *p* and/or *q*.

(a) log*a* 10

(b) log*a* 8

(c) log*a* 2.5

(Total 6 marks)

**17.** Consider the expansion of the expression (*x*3 − 3*x*)6.

(a) Write down the number of terms in this expansion.

(b) Find the term in *x*12.

(Total 6 marks)

**18.** One of the terms of the expansion of (*x* + 2*y*)10 is *ax*8 *y*2. Find the value of *a*.

(Total 6 marks)

**19.** The first four terms of a sequence are 18, 54, 162, 486.

(a) Use all four terms to show that this is a geometric sequence.

(2)

(b) (i) Find an expression for the *n*th term of this geometric sequence.

(ii) If the *n*th term of the sequence is 1062 882, find the value of *n*.

(4)

(Total 6 marks)

**20.** (a) Write down the first three terms of the sequence *un* = 3*n*, for *n* 1.

(1)

(b) Find

(i) 

(ii) .

(5)

(Total 6 marks)

**21.** (a) Expand  in terms of e.

(4)

(b) Express  +  as the sum of three terms.

(2)

(Total 6 marks)

**22.** (a) Let log*c* 3 = *p* and log*c* 5 = *q*. Find an expression in terms of *p* and *q* for

(i) log *c* 15;

(ii) log *c* 25.

(b) Find the value of *d* if log *d* 6 = .

(Total 6 marks)

**23.** Consider the infinite geometric series 405 + 270 + 180 +....

(a) For this series, find the common ratio, giving your answer as a fraction in its simplest form.

(b) Find the fifteenth term of this series.

(c) Find the **exact** value of the sum of the infinite series.

(Total 6 marks)

**24.** Let ln *a* = *p*, ln *b* = *q*. Write the following expressions in terms of *p* and *q*.

(a) ln *a*3*b*

(b) ln 

(Total 6 marks)

**25.** A theatre has 20 rows of seats. There are 15 seats in the first row, 17 seats in the second row, and each successive row of seats has two more seats in it than the previous row.

(a) Calculate the number of seats in the 20th row.

(b) Calculate the **total** number of seats.

(Total 6 marks)

**26.** A sum of $5000 is invested at a compound interest rate of 6.3% per annum.

(a) Write down an expression for the value of the investment after *n* full years.

(b) What will be the value of the investment at the end of five years?

(c) The value of the investment will exceed $10000 after *n* full years,

(i) Write down an inequality to represent this information.

(ii) Calculate the minimum value of *n*.

(Total 6 marks)

**27.** Let *Sn* be the sum of the first *n* terms of an arithmetic sequence, whose first three terms are *u*1, *u*2 and *u*3. It is known that *S*1 = 7, and *S*2 = 18.

(a) Write down *u*1.

(b) Calculate the common difference of the sequence.

(c) Calculate *u*4.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:(a) ..................................................................(b) ..................................................................(c) .................................................................. |

(Total 6 marks)

**28.** Consider the expansion of (*x*2 – 2)5.

(a) Write down the number of terms in this expansion.

(b) The first four terms of the expansion in descending powers of *x* are

*x*10 – 10*x*8 + 40*x*6 + *Ax*4 + ...

 Find the value of *A*.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers:*(a) ..................................................................(b) .................................................................. |

(Total 6 marks)

**29.** Find the **exact** solution of the equation 92*x* = 27(1–*x*).

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer*:....…………………………………….......... |

(Total 6 marks)

**30.** (a) Given that log3 *x* – log3 (*x* – 5) = log3 *A*, express *A* in terms of *x*.

(b) Hence or otherwise, solve the equation log3 *x* – log3 (*x* – 5) = 1.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:(a) ..................................................................(b) .................................................................. |

(Total 6 marks)

**31.** Given that  = *p* +  where *p* and *q* are integers, find

(a) *p*;

(b) *q*.



(Total 6 marks)

**32.** The first term of an infinite geometric sequence is 18, while the third term is 8. There are two possible sequences. Find the sum of each sequence.



(Total 6 marks)

**33.** Let *p* = log10 *x*, *q* = log10 *y* and *r* = log10 *z*.

 Write the expression log10 in terms of *p*, *q* and *r*.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer*:………………………………………….. |

(Total 6 marks)

**34.** The following table shows four series of numbers. One of these series is geometric, one of the series is arithmetic and the other two are neither geometric nor arithmetic.

(a) Complete the table by stating the type of series that is shown.

|  |  |  |
| --- | --- | --- |
| Series |  | Type of series |
| (i) | 111111111111111… |  |
| (ii) | 1… |  |
| (iii) | 0.90.8750.850.8250.8… |  |
| (iv) |  |  |

(b) The geometric series can be summed to infinity. Find this sum.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer:*(b) ………………………………………….. |

(Total 6 marks)

**35.** When the expression (2 + *ax*)10 is expanded, the coefficient of the term in *x*3 is 414 720. Find the value of *a*.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer*:………………………………………….. |

(Total 6 marks)

**36.** Find the term containing *x*3 in the expansion of (2 – 3*x*)8.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer*:…………………………………………........ |

(Total 6 marks)

**37.** Let *a* = log *x*, *b* = log *y*, and *c* = log *z*.

 Write log  in terms of *a*, *b* and *c*.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer*:…………………………………………........ |

(Total 6 marks)

**38.** Gwendolyn added the multiples of 3, from 3 to 3750 and found that

3 + 6 + 9 + … + 3750 = *s*.

 Calculate *s*.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer*:.................................................................. |

(Total 6 marks)

**39.** Find the term containing *x*10 in the expansion of (5 + 2*x*2)7.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer*:.................................................................. |

(Total 6 marks)

**40.** The number of hours of sleep of 21 students are shown in the frequency table below.

|  |  |
| --- | --- |
| **Hours of sleep** | **Number of students** |
| 4 | 2 |
| 5 | 5 |
| 6 | 4 |
| 7 | 3 |
| 8 | 4 |
| 10 | 2 |
| 12 | 1 |

 Find

(a) the median;

(b) the lower quartile;

(c) the interquartile range.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:(a) ..................................................................(b) ..................................................................(c) .................................................................. |

(Total 6 marks)

**41.** Given that log5 *x* = *y*, express each of the following in terms of *y*.

(a) log5 *x*2

(b) log5

(c) log25 *x*

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:(a) ..................................................................(b) ..................................................................(c) .................................................................. |

(Total 6 marks)

**42.** Complete the following expansion.

(2 + *ax*)4 = 16 + 32*ax* + …

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer*:.................................................................. |

(Total 6 marks)

**43.** Arturo goes swimming every week. He swims 200 metres in the first week. Each week he swims 30 metres more than the previous week. He continues for one year (52 weeks).

(a) How far does Arturo swim in the final week?

(b) How far does he swim altogether?

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:(a) ..................................................................(b) .................................................................. |

(Total 6 marks)

**44.** In an arithmetic sequence, the first term is –2, the fourth term is 16, and the *n*th term is 11 998.

(a) Find the common difference *d*.

(b) Find the value of *n*.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:(a) ..................................................................(b) .................................................................. |

(Total 6 marks)

**45.** Consider the expansion of 

(a) How many terms are there in this expansion?

(b) Find the constant term in this expansion.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:(a) ..................................................................(b) .................................................................. |

(Total 6 marks)

**46.** Solve the equation log27 *x* = 1 – log27 (*x* – 0.4).

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer*:...................................................................... |

(Total 6 marks)

**47.** Consider the following statements

A: log10 (10*x*) > 0.

B: –0.5  cos (0.5*x*)  0.5.

C: –   arctan *x*  .

(a) Determine which statements are true for all real numbers *x*. Write your answers (yes or no) in the table below.

|  |  |  |
| --- | --- | --- |
| Statement | (a) Is the statement true for all real numbers *x*? (Yes/No) | (b) If not true, example |
| A |  |  |
| B |  |  |
| C |  |  |

(b) If a statement is not true for all *x*, complete the last column by giving an example of one value of *x* for which the statement is false.

|  |
| --- |
| *Working:* |

(Total 6 marks)

**48.** Find the coefficient of *x*3 in the expansion of (2 – *x*)5.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer*:...................................................................... |

(Total 6 marks)

**49.** Use the binomial theorem to complete this expansion.

(3*x* + 2*y*)4 = 81*x*4 + 216*x*3 *y +...*

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer:*....................................................................... |

(Total 4 marks)

**50.** The first three terms of an arithmetic sequence are 7, 9.5, 12.

(a) What is the 41st term of the sequence?

(b) What is the sum of the first 101 terms of the sequence?

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:(a) ..................................................................(b) .................................................................. |

(Total 4 marks)

**51.** Solve the equation log9 81 + log9  + log9 3 = log9 *x.*

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer:*....................................................................... |

(Total 4 marks)

**52.** Consider the binomial expansion 

(a) By substituting *x* = 1 into both sides, or otherwise, evaluate 

(b) Evaluate .

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:(a) ..................................................................(b) .................................................................. |

(Total 4 marks)

**53.** Let log10*P* = *x* , log10*Q* = *y* and log10*R* = *z*. Express  in terms of *x* , *y* and *z*.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer:*.................................................................... |

(Total 4 marks)

**54.** Each day a runner trains for a 10 km race. On the first day she runs 1000 m, and then increases the distance by 250 m on each subsequent day.

(a) On which day does she run a distance of 10 km in training?

(b) What is the total distance she will have run in training by the end of that day? Give your answer exactly.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:(a) ..................................................................(b) .................................................................. |

(Total 4 marks)

**55.** Determine the constant term in the expansion of 

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer:*.................................................................... |

(Total 4 marks)

**56.** In an arithmetic sequence, the first term is 5 and the fourth term is 40. Find the second term.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer:*...................................................................... |

(Total 4 marks)

**57.** If log*a* 2 = *x* and log*a* 5 = *y,* find in terms of *x* and *y,* expressions for

(a) log2 5;

(b) loga 20.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:(a) ..................................................................(b) .................................................................. |

(Total 4 marks)

**58.** Find the sum of the infinite geometric series



|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer:*...................................................................... |

(Total 4 marks)

**59.** Find the coefficient of *a*5*b*7in the expansion of (*a* + *b*)12*.*

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer:*...................................................................... |

(Total 4 marks)

**60.** $1000 is invested at the beginning of each year for 10 years.

 The rate of interest is fixed at 7.5% per annum. Interest is compounded annually.

 Calculate, giving your answers to the nearest dollar

(a) how much the first $1000 is worth at the end of the ten years;

(b) the total value of the investments at the end of the ten years.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answers*:(a) ..................................................................(b) .................................................................. |

(Total 4 marks)

**61.** Find the sum of the arithmetic series

17 + 27 + 37 +...+ 417.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer*:......................................................................... |

(Total 4 marks)

**62.** Solve the equation 9*x*–1 = 

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer:*...................................................................... |

(Total 4 marks)

**63.** Find the coefficient of *x*5 in the expansion of (3*x* – 2)8.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer:*...................................................................... |

(Total 4 marks)

**64.** An arithmetic series has five terms. The first term is 2 and the last term is 32. Find the sum of the series.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer:*...................................................................... |

(Total 4 marks)

**65.** Find the coefficient of *a*3*b*4 in the expansion of (5*a* + *b*)7.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer:*...................................................................... |

(Total 4 marks)

**66.** Solve the equation 43*x*–1 = 1.5625 × 10–2.

|  |  |
| --- | --- |
| *Working:* |  |
|  | *Answer:*...................................................................... |

(Total 4 marks)