SIMPLIFYING TRIGONOMETRIC EXPRESSIONS

Example 10   ◀ၒ **Self Tutor**

Simplify:

    a $3\cos\theta + 4\cos\theta$ **b** $\tan\alpha - 3\tan\alpha$

$3a + 4a = 7a$

$7\cos\theta$

$-2\tan\alpha$

To simplify complicated trigonometric expressions, we often use the identities:

1a= $\sin^2\theta = 1 - \cos^2\theta$

① $\sin^2\theta + \cos^2\theta = 1$ $(\sin\theta)^2$

1b= $\cos^2\theta = 1 - \sin^2\theta$

② $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $= \sin^2\theta$

Example 11   ◀Ð **Self Tutor**

Simplify: GCF

a $2 - 2\sin^2\theta$ b $\cos^2\theta\sin\theta + \sin^3\theta$

$2(1 - \sin^2\theta)$ | $\sin\theta(\cos^2\theta + \sin^2\theta)$

$2\cos^2\theta$ | $= \sin\theta$

Example 12 ◄⑴ **Self Tutor**

Expand and simplify: $(\cos\theta - \sin\theta)^2$

$$(a-b)^2 =$$

$$\cos\theta = a$$
$$\sin\theta = b$$

$$(a-b)(a-b) = a^2 - 2ab + b^2$$

$$(\cos\theta - \sin\theta)^2 = \cos^2\theta - 2\sin\theta$$

$$\cos\theta + \sin2\theta$$

$$\cos^2\theta$$
$$(\cos\theta)^2$$

$$= \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta$$

$$= 1 - 2\sin\theta\cos\theta$$

Not →
Now → $= 1 - \sin2\theta$

EXERCISE 11C.1

1 Simplify:

   a $\sin\theta + \sin\theta$ **b** $2\cos\theta + \cos\theta$ **c** $3\sin\theta - \sin\theta$

   d $3\sin\theta - 2\sin\theta$ **e** $\tan\theta - 3\tan\theta$ **f** $2\cos^2\theta - 5\cos^2\theta$

2 Simplify:

   a $3\sin^2\theta + 3\cos^2\theta$ **b** $-2\sin^2\theta - 2\cos^2\theta$ **c** $-\cos^2\theta - \sin^2\theta$

   d $3 - 3\sin^2\theta$ **e** $4 - 4\cos^2\theta$ **f** $\cos^3\theta + \cos\theta\sin^2\theta$

   g $\cos^2\theta - 1$ **h** $\sin^2\theta - 1$ **i** $2\cos^2\theta - 2$

   j $\dfrac{1 - \sin^2\theta}{\cos^2\theta}$ **k** $\dfrac{1 - \cos^2\theta}{\sin\theta}$ **l** $\dfrac{\cos^2\theta - 1}{-\sin\theta}$

3 Simplify:

   a $3\tan x - \dfrac{\sin x}{\cos x}$ **b** $\dfrac{\sin^2 x}{\cos^2 x}$ **c** $\tan x \cos x$

   d $\dfrac{\sin x}{\tan x}$ **e** $3\sin x + 2\cos x \tan x$ **f** $\dfrac{2\tan x}{\sin x}$

4 Expand and simplify if possible:

   a $(1 + \sin\theta)^2$ **b** $(\sin\alpha - 2)^2$ **c** $(\tan\alpha - 1)^2$

   d $(\sin\alpha + \cos\alpha)^2$ **e** $(\sin\beta - \cos\beta)^2$ **f** $-(2 - \cos\alpha)^2$

5 Expand and simplify: $(\sin x + \tan x)(\sin x - \tan x)$

FACTORISING TRIGONOMETRIC EXPRESSIONS

Example 13

Factorise: **a** $\cos^2 \alpha - \sin^2 \alpha$ **b** $\tan^2 \theta - 3\tan\theta + 2$

difference
btw Squares.

$$(a^2 - b^2) = (a-b)(a+b)$$

$$a = \cos\alpha$$

$$b = \sin\alpha$$

$$= (\cos\alpha - \sin\alpha)(\cos\alpha + \sin\alpha)$$

$x^2 - 3x + 2$ $x = \tan\theta$

$(x-2)(x-1)$ $\tan^2\theta - 3\tan\theta + 2$

$$= (\tan\theta - 2)(\tan\theta - 1)$$

Example 14 ◀)) **Self Tutor**

Simplify:

a $\dfrac{2 - 2\cos^2\theta}{1 + \cos\theta}$

b $\dfrac{\cos\theta - \sin\theta}{\cos^2\theta - \sin^2\theta}$

$$\dfrac{2(1-\cos^2\theta)}{1+\cos\theta}$$

$$\dfrac{2\sin^2\theta}{1+\cos\theta} \qquad \dfrac{2[(1-\cos\theta)(1+\cos\theta)]}{(1+\cos\theta)}$$

$\times$

$$2(1-\cos\theta)$$

$\checkmark$ more simplified

$$\dfrac{\cos\theta - \sin\theta}{\cos^2\theta - \sin^2\theta}$$

$$= \dfrac{(\cos\theta - \sin\theta)}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$$

$$= \dfrac{1}{\cos\theta + \sin\theta}$$

EXERCISE 11C.2

1 Factorise:

    a $1 - \sin^2 \theta$ **b** $\sin^2 \alpha - \cos^2 \alpha$ **c** $\tan^2 \alpha - 1$

    d $2\sin^2 \beta - \sin \beta$ **e** $2\cos \phi + 3\cos^2 \phi$ **f** $3\sin^2 \theta - 6\sin \theta$

    g $\tan^2 \theta + 5\tan \theta + 6$ **h** $2\cos^2 \theta + 7\cos \theta + 3$ **i** $6\cos^2 \alpha - \cos \alpha - 1$

2 Simplify:

    a $\dfrac{1 - \sin^2 \alpha}{1 - \sin \alpha}$ **b** $\dfrac{\tan^2 \beta - 1}{\tan \beta + 1}$ **c** $\dfrac{\cos^2 \phi - \sin^2 \phi}{\cos \phi + \sin \phi}$

    d $\dfrac{\cos^2 \phi - \sin^2 \phi}{\cos \phi - \sin \phi}$ **e** $\dfrac{\sin \alpha + \cos \alpha}{\sin^2 \alpha - \cos^2 \alpha}$ **f** $\dfrac{3 - 3\sin^2 \theta}{6\cos \theta}$

3 Show that:

    a $(\cos \theta + \sin \theta)^2 + (\cos \theta - \sin \theta)^2$ simplifies to 2

    b $(2\sin \theta + 3\cos \theta)^2 + (3\sin \theta - 2\cos \theta)^2$ simplifies to 13

    c $(1 - \cos \theta)\left(1 + \dfrac{1}{\cos \theta}\right)$ simplifies to $\tan \theta \sin \theta$

    d $\left(1 + \dfrac{1}{\sin \theta}\right)(\sin \theta - \sin^2 \theta)$ simplifies to $\cos^2 \theta$

    e $\dfrac{\sin \theta}{1 + \cos \theta} + \dfrac{1 + \cos \theta}{\sin \theta}$ simplifies to $\dfrac{2}{\sin \theta}$

    f $\dfrac{\sin \theta}{1 - \cos \theta} - \dfrac{\sin \theta}{1 + \cos \theta}$ simplifies to $\dfrac{2}{\tan \theta}$

    g $\dfrac{1}{1 - \sin \theta} + \dfrac{1}{1 + \sin \theta}$ simplifies to $\dfrac{2}{\cos^2 \theta}$.