**1.** Let *f*(*x*) =3*x*2. The graph of *f* is translated 1 unit to the right and 2 units down.
The graph of *g* is the image of the graph of *f* after this translation.

(a) Write down the coordinates of the vertex of the graph of *g*.

(2)

(b) Express *g* in the form *g*(*x*)= 3(*x* – *p*)2 + *q*.

(2)

 The graph of *h* is the reflection of the graph of *g* in the *x*-axis.

(c) Write down the coordinates of the vertex of the graph of *h*.

(2)

(Total 6 marks)

**2.** Let *f*(*x*)= log3  + log3 16 – log3 4, for *x* > 0.

(a) Show that *f*(*x*) = log32*x*.

(2)

(b) Find the value of *f*(0.5)and of *f*(4.5).

(3)

 The function *f* can also be written in the form *f*(*x*) = .

(c) (i) Write down the value of *a* and of *b*.

(ii) Hence on graph paper, **sketch** the graph of *f*, for –5 ≤ *x* ≤ 5, –5 ≤ *y* ≤ 5, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote.

(6)

(d) Write down the value of *f*–1(0).

(1)

 The point A lies on the graph of *f*. At A, *x* = 4.5.

(e) On your diagram, sketch the graph of *f*–1, noting clearly the image of point A.

(4)

(Total 16 marks)

**3.** Consider *f*(*x*) = 2 – *x*2, for –2 ≤ *x* ≤ 2and *g*(*x*) = sin e*x*, for –2 ≤ *x* ≤ 2. The graph of *f* is given below.

 

(a) On the diagram above, sketch the graph of *g*.

(3)

(b) Solve *f*(*x*)= *g*(*x*).

(2)

(c) Write down the set of values of *x* such that *f*(*x*) *> g*(*x*).

(2)

(Total 7 marks)

**4.** The number of bacteria, *n*, in a dish, after *t* minutes is given by *n* = 800e013*t*.

(a) Find the value of *n* when *t* = 0.

(2)

(b) Find the rate at which *n* is increasing when *t* = 15.

(2)

(c) After *k* minutes, the rate of increase in *n* is greater than 10 000 bacteria per minute. Find the least value of *k*, where *k*  .

(4)

(Total 8 marks)

**5.** The quadratic equation *kx*2 + (*k* – 3)*x* + 1 = 0 has two equal real roots.

(a) Find the possible values of *k*.

(5)

(b) **Write down** the values of *k* for which *x*2+ (*k* – 3)*x* + *k* = 0 has two equal real roots.

(2)

(Total 7 marks)

**6.** Let *f* (*x*) = 3*x* – e*x*–2 – 4, for –1  *x*  5.

(a) Find the *x*-intercepts of the graph of *f*.

(3)

(b) On the grid below, sketch the graph of *f*.



(3)

(c) Write down the gradient of the graph of *f* at *x* = 2.

(1)

(Total 7 marks)

**7.** A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After *n* years the number of taxis, *T*, in the city is given by

 *T* = 280  1.12*n*.

(a) (i) Find the number of taxis in the city at the end of 2005.

(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000.

(6)

(b) At the end of 2000 there were 25 600 people in the city who used taxis. After *n* years the number of people, *P*, in the city who used taxis is given by

 *P* = 

(i) Find the value of *P* at the end of 2005, giving your answer to the nearest whole number.

(ii) After seven complete years, will the value of *P* be double its value at the end of 2000? Justify your answer.

(6)

(c) Let *R* be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if *R*  70.

(i) Find the value of *R* at the end of 2000.

(ii) After how many complete years will the city first reduce the number of taxis?

(5)

(Total 17 marks)

**8.** Let *f*(*x*) = 2*x*2 + 4*x* – 6.

(a) Express *f*(*x*) in the form *f*(*x*) = 2(*x* – *h*)2 + *k*.

(3)

(b) Write down the equation of the axis of symmetry of the graph of *f.*

(1)

(c) Express *f*(*x*) in the form *f*(*x*) = 2(*x* – *p*)(*x* – *q*).

(2)

(Total 6 marks)

**9.** The function *f* is defined by *f*(*x*) = , for –3 < *x* < 3.

(a) On the grid below, sketch the graph of *f*.

 

(2)

(b) Write down the equation of each vertical asymptote.

(2)

(c) Write down the range of the function *f*.

(2)

(Total 6 marks)

**10.** The functions *f* and *g* are defined by *f* : *x*  3*x*, *g* : *x*  *x* + 2.

(a) Find an expression for (*f* ° *g*)(*x*).

(2)

(b) Find *f*–1(18) + *g*–1(18).

(4)

(Total 6 marks)

**11.** Let *f* (*x*) = , where *p*, *q* +.

Part of the graph of *f*, including the asymptotes, is shown below.



(a) The equations of the asymptotes are *x* =1, *x* = −1, *y* = 2. Write down the value of

(i) *p*;

(ii) *q*.

(2)

(b) Let *R* be the region bounded by the graph of *f*, the *x*-axis, and the *y*-axis.

(i) Find the negative *x*-intercept of *f*.

(ii) Hence find the volume obtained when *R* is revolved through 360 about the *x*-axis.

(7)

(c) (i) Show that *f* ′ (*x*) = .

(ii) Hence, show that there are no maximum or minimum points on the graph of *f*.

(8)

(d) Let *g* (*x*) = *f* ′ (*x*). Let *A* be the area of the region enclosed by the graph of g and the *x*-axis, between *x* = 0 and *x* = *a*, where *a*  0. Given that *A* = 2, find the value of *a*.

(7)

(Total 24 marks)

**12.** Two weeks after its birth, an animal weighed 13 kg. At 10 weeks this animal weighed 53 kg. The increase in weight each week is constant.

(a) Show that the relation between *y*, the weight in kg, and *x*, the time in weeks, can be written as *y* = 5*x* + 3

(2)

(b) Write down the weight of the animal at birth.

(1)

(c) Write down the weekly increase in weight of the animal.

(1)

(d) Calculate how many weeks it will take for the animal to reach 98 kg.

(2)

(Total 6 marks)

**13.** Consider the function *f* (*x*) =  + 8, *x*  10.

(a) Write down the **equation** of

(i) the vertical asymptote;

(ii) the horizontal asymptote.

(2)

(b) Find the

(i) *y*-intercept;

(ii) *x*-intercept.

(2)

(c) Sketch the graph of *f* , clearly showing the above information.

(4)

(d) Let *g* (*x*) = , *x*  0.

The graph of *g* is transformed into the graph of *f* using two transformations.

 The first is a translation with vector  Give a full geometric description of the second transformation.

(2)

(Total 10 marks)

**14.** There were 1420 doctors working in a city on 1 January 1994. After *n* years the number of doctors, *D*, working in the city is given by

*D* = 1420 + 100*n.*

(a) (i) How many doctors were there working in the city at the start of 2004?

(ii) In what year were there first more than 2000 doctors working in the city?

(3)

 At the beginning of 1994 the city had a population of 1.2 million. After *n* years, the population, *P*, of the city is given by

*P* = 1 200 000 (1.025)*n*.

(b) (i) Find the population *P* at the beginning of 2004.

(ii) Calculate the percentage growth in population between 1 January 1994 and 1 January 2004.

(iii) In what year will the population first become greater than 2 million?

(7)

(c) (i) What was the average number of people per doctor at the beginning of 1994?

(ii) After how many **complete** years will the number of people per doctor first fall below 600?

(5)

(Total 15 marks)

**15.** Michele invested 1500 francs at an annual rate of interest of 5.25 percent,
compounded annually.

(a) Find the value of Michele’s investment after 3 years. Give your answer to the nearest franc.

(3)

(b) How many complete years will it take for Michele’s initial investment to double in value?

(3)

(c) What should the interest rate be if Michele’s initial investment were to double in value in 10 years?

(4)

(Total 10 marks)

**16.** A ball is thrown vertically upwards into the air. The height, *h* metres, of the ball above the ground after *t* seconds is given by

*h* = 2 + 20*t* – 5*t*2, *t*  0

(a) Find the **initial** height above the ground of the ball (that is, its height at the instant when it is released).

(2)

(b) Show that the height of the ball after one second is 17 metres.

(2)

(c) At a later time the ball is **again** at a height of 17 metres.

(i) Write down an equation that *t* must satisfy when the ball is at a height of 17 metres.

(ii) Solve the equation **algebraically**.

(4)

(d) (i) Find .

(ii) Find the **initial** velocity of the ball (that is, its velocity at the instant when it is released).

(iii) Find **when** the ball reaches its maximum height.

(iv) Find the maximum height of the ball.

(7)

(Total 15 marks)

**17.** Initially a tank contains 10 000 litres of liquid. At the time *t* = 0 minutes a tap is opened, and liquid then flows out of the tank. The volume of liquid, *V* litres, which remains in the tank after *t* minutes is given by

*V* = 10 000 (0.933*t*).

(a) Find the value of *V* after 5 minutes.

(1)

(b) Find how long, to the nearest second, it takes for half of the initial amount of liquid to flow out of the tank.

(3)

(c) The tank is regarded as effectively empty when 95% of the liquid has flowed out.
Show that it takes almost three-quarters of an hour for this to happen.

(3)

(d) (i) Find the value of 10 000 – *V* when *t =* 0.001 minutes.

(ii) Hence or otherwise, estimate the initial flow rate of the liquid.
Give your answer in litres per minute, correct to two significant figures.

(3)

(Total 10 marks)