IB Maths SL The Binomial Expansion Practice Problems Mr. W Name____________________

Remember to show all necessary reasoning! Due to a desire to use less copy paper, Mr. W has probably not given you enough space for most problems, so doing this on separate paper is recommended.

1. Consider the expansion of $(x + 2)^{11}$.

(a) Write down the number of terms in this expansion.

(1)

(b) Find the term containing x^2.

(4)
(Total 5 marks)

2. (a) Expand $(2 + x)^4$ and simplify your result.

(3)

(b) Hence, find the term in x^2 in $(2 + x)^4\left(1+\frac{1}{x^2}\right)$.

(3)
(Total 6 marks)

3. Find the term in x^4 in the expansion of $\left(3x^2 - \frac{2}{x}\right)^5$.

(Total 6 marks)

4. The fifth term in the expansion of the binomial $(a + b)^n$ is given by $\binom{10}{4}p^6(2q)^4$.

(a) Write down the value of n.

(1)

(b) Write down a and b, in terms of p and/or q.

(2)

(c) Write down an expression for the sixth term in the expansion.

(3)
(Total 6 marks)

5. (OPTIONAL CHALLENGE PROBLEM!!) Let $f(x) = x^3 - 4x + 1$.

(a) Expand $(x + h)^3$.

(2)

(b) Use the formula $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to show that the derivative of $f(x)$ is $3x^2 - 4$.

(4)

(c) The tangent to the curve of f at the point P(1, –2) is parallel to the tangent at a point Q. Find the coordinates of Q.

(4)

(d) The graph of f is decreasing for $p < x < q$. Find the value of p and of q.

(3)

(e) Write down the range of values for the gradient of f.

(2)
(Total 15 marks)

6. Find the term in x^3 in the expansion of $\left(\frac{2}{3}x - 3\right)^8$.

(Total 5 marks)

7. (a) Expand $(x - 2)^4$ and simplify your result.

(3)

(b) Find the term in x^3 in $(3x + 4)(x - 2)^4$.

(3)
(Total 6 marks)

8. Consider the expansion of the expression $(x^3 - 3x)^6$.

(a) Write down the number of terms in this expansion.

(b) Find the term in x^{12}.

(Total 6 marks)

9. One of the terms of the expansion of $(x + 2y)^{10}$ is $ax^8 y^2$. Find the value of a.

(Total 6 marks)

10. (a) Expand $\left(\mathrm{e} + \frac{1}{\mathrm{e}}\right)^4$ in terms of e.

(4)

(b) Express $\left(\mathrm{e} + \frac{1}{\mathrm{e}}\right)^4 + \left(\mathrm{e} - \frac{1}{\mathrm{e}}\right)^4$ as the sum of three terms.

(2)
(Total 6 marks)

WORKED OUT SOLUTIONS

1. (a) 12 terms A1 N1 1

(b) evidence of binomial expansion (M1)

e.g. $\binom{n}{r} a^{n-r} b^{r}$, an attempt to expand, Pascal's triangle

evidence of choosing correct term (A1)

e.g. 10th term, $r = 9$, $\binom{11}{9}(x)^2 (2)^9$

correct working A1

e.g. $\binom{11}{9}$ $(x)^2 (2)^9$, 55×2^9

$28160x^2$ A1 N2 4

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2. (a) evidence of expanding M1

e.g. $2^4 + 4(2^3)x + 6(2^2)x^2 + 4(2)x^3 + x^4$, $(4 + 4x + x^2)(4 + 4x + x^2)$

$(2 + x)^4 = 16 + 32x + 24x^2 + 8x^3 + x^4$ A2 N2

(b) finding coefficients 24 and 1 (A1)(A1)

term is $25x^2$ A1 N3

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3. evidence of substituting into binomial expansion (M1)

e.g. $a^5 + \binom{5}{1} a^4 b + \binom{5}{2} a^3 b^2 + \ldots$

identifying correct term for x^4 (M1)

evidence of calculating the factors, in any order A1A1A1

e.g. $\binom{5}{2}$, $27x^6, \frac{4}{x^2}$; $10(3x^2)^3\left(\frac{-2}{x}\right)^2$

Note: *Award A1 for each correct factor.*

term = $1080x^4$ A1 N2

Note: *Award M1M1A1A1A1A0 for 1080 with working shown.*

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4. (a) $n = 10$ A1 N1

(b) $a = p$, $b = 2q$ (or $a = 2q$, $b = p$) A1A1 N1N1

(c) $\binom{10}{5} p^5 (2q)^5$ A1A1A1 N3

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5. (a) attempt to expand (M1)

$(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$ A1 N2

(b) evidence of substituting x + h (M1)

correct substitution A1

e.g. $f'(x) = \lim_{h\to 0}\frac{(x+h)^3 - 4(x+h)+1-(x^3-4x+1)}{h}$

simplifying A1

e.g. $\frac{(x^3+3x^2h+3xh^2+h^2-4x-4h+1-x^3+4x-1)}{h}$

factoring out h A1

e.g. $\frac{h(3x^2+3xh+h^2-4)}{h}$

$f'(x) = 3x^2 - 4$ AG N0

(c) $f'(1) = -1$ (A1)

setting up an appropriate equation M1

e.g. $3x^2 - 4 = -1$

at Q, $x = -1$, $y = 4$ (Q is $(-1, 4)$) A1A1 N3

(d) recognizing that f is decreasing when $f'(x) < 0$ R1

correct values for p and q (but do not accept $p = 1.15$, $q = -1.15$) A1A1 N1N1

e.g. $p = -1.15$, $q = 1.15$; $\pm\frac{2}{\sqrt{3}}$; an interval such as $-1.15 \le x \le 1.15$

(e) $f'(x) \ge -4$, $y \ge -4$, $[-4, \infty[$ A2 N2

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6. evidence of using binomial expansion (M1)

e.g. selecting correct term, $a^8b^0 + \binom{8}{1}a^7b + \binom{8}{2}a^6b^2 + \ldots$

evidence of calculating the factors, in any order A1A1A1

e.g. 56, $\frac{2^3}{3^3}, -3^5, \binom{8}{5}\left(\frac{2}{3}x\right)^3(-3)^5$

$-4032x^3$ (accept $= -4030x^3$ to 3 s.f.) A1 N2

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7. (a) evidence of expanding M1

e.g. $(x - 2)^4 = x^4 + 4x^3(-2) + 6x^2(-2)^2 + 4x(-2)^3 + (-2)^4$

$(x - 2)^4 = x^4 - 8x^3 + 24x^2 - 32x + 16$ A2 N2

(b) finding coefficients, 3×24 (= 72), $4 \times (-8)$(= -32) (A1)(A1)

term is $40x^3$ A1 N3

[6]

8. (a) 7 terms A1 N1

(b) A valid approach (M1)

Correct term **chosen** $\binom{6}{3}\left(x^3\right)^3(-3x)^3$ A1

Calculating $\binom{6}{3}=20, (-3)^3=-27$ (A1)(A1)

Term is $-540x^{12}$ A1 N3

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9. Identifying the required term (seen anywhere) M1

eg $\binom{10}{8}\times 2^2$

$\binom{10}{8}$ = 45 (A1)

$4y^2$, 2 × 2, 4 (A2)

a = 180 A2 N4

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10. (a) For finding second, third and fourth terms correctly(A1)(A1)(A1)

Second term $\binom{4}{1}e^3\left(\frac{1}{e}\right)$, third term $\binom{4}{1}e^2\left(\frac{1}{e}\right)^2$,

fourth term $\binom{4}{1}e\left(\frac{1}{e}\right)^3$

For finding first and last terms, **and** adding them to **their** three terms (A1)

$$\left(e+\frac{1}{e}\right)^4=\binom{4}{0}e^4+\binom{4}{1}e^3\left(\frac{1}{e}\right)+\binom{4}{2}e^2\left(\frac{1}{e}\right)^2+\binom{4}{3}e\left(\frac{1}{e}\right)^3+\binom{4}{4}\left(\frac{1}{e}\right)^4$$

$$\left(e+\frac{1}{e}\right)^4=e^4+4e^3\left(\frac{1}{e}\right)+6e^2\left(\frac{1}{e}\right)^2+4e\left(\frac{1}{e}\right)^3+\left(\frac{1}{e}\right)^4$$

$$\left(=e^4+4e^2+6+\frac{4}{e^2}+\frac{1}{e^4}\right)$$ N4

(b) $\left(e-\frac{1}{e}\right)^4 = e^4 - 4e^3\left(\frac{1}{e}\right) + 6e^2\left(\frac{1}{e}\right)^2 - 4e\left(\frac{1}{e}\right)^3 + \left(\frac{1}{e}\right)^4$

$\left(= e^4 - 4e^2 + 6 - \frac{4}{e^2} + \frac{1}{e^4}\right)$ (A1)

Adding gives $2e^4 + 12 + \frac{2}{e^4}$

$\left(\text{accept } 2\binom{4}{0}e^4 + 2\binom{4}{2}e^2\left(\frac{1}{e}\right)^2 + 2\binom{4}{4}\left(\frac{1}{e}\right)^4\right)$ A1 N2

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