**1.** (a) evidence of choosing cosine rule (M1)

*e.g. a*2 + *b*2 – 2*ab* cos *C*

correct substitution A1

*e.g.* 72 + 92 – 2(7)(9) cos 120º

AC =13.9 (=) A1 N2 3



(b) **METHOD 1**

evidence of choosing sine rule (M1)

*e.g.*



correct substitution A1

*e.g.*



A1 N2 3



**METHOD 2**

evidence of choosing cosine rule (M1)

*e.g.*



correct substitution A1

*e.g.*



 A1 N2 3



[6]

**2.** (a) **METHOD 1**

evidence of recognizing the amplitude is the radius (M1)

*e.g.* amplitude is half the diameter

A1



*a* = 4 AG N02

**METHOD 2**

evidence of recognizing the maximum height (M1)

*e.g. h* = 6, *a* sin *bt* + 2 = 6

correct reasoning

*e.g. a* sin *bt* = 4 and sin *bt* has amplitude of 1 A1

*a* = 4 AG N02

(b) **METHOD 1**

period = 30 (A1)

A1



AG N02



**METHOD 2**

correct equation (A1)

*e.g.* 2 = 4 sin 30*b* + 2, sin 30*b* = 0

30*b* = 2π A1

AG N02



(c) recognizing *h*′(*t*) = –0.5 (seen anywhere) R1

attempting to solve (M1)

*e.g.* sketch of *h*′, finding *h*′

correct work involving *h*′ A2

*e.g.* sketch of *h*′ showing intersection, –0.5 =



*t* = 10.6, *t* = 19.4 A1A1N3 **6**

(d) **METHOD 1**

valid reasoning for **their** conclusion (seen anywhere) R1

*e.g. h*(*t*) < 0 so underwater; *h*(*t*) > 0 so not underwater

evidence of substituting into *h* (M1)

*e.g. h*(19.4),



correct calculation A1

*e.g. h*(19.4) = –1.19

correct statement A1 N04

*e.g.* the bucket is underwater, yes

**METHOD 2**

valid reasoning for **their** conclusion (seen anywhere) R1

*e.g. h*(*t*) < 0 so underwater; *h*(*t*) > 0 so not underwater

evidence of valid approach (M1)

*e.g.* solving *h*(*t*) = 0, graph showing region below *x*-axis

correct roots A1

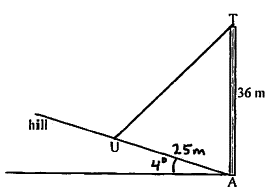
*e.g.* 17.5, 27.5

correct statement A1N04

*e.g.* the bucket is underwater, yes

[14]

**3.** (a)  
 A1A1A1N33



**Note:** Award A1for labelling 4° with horizontal, A1for labelling [AU] 25 metres, A1for drawing [TU].

(b) TÂU = 86º (A1)

evidence of choosing cosine rule (M1)

correct substitution A1

*e.g. x*2 = 252 + 362 – 2(25)(36) cos 86º

*x* = 42.4 A1N34

[7]

**4.** (a) evidence of finding height, *h* (A1)

*e.g.* sin *θ* = , 2 sin *θ*



evidence of finding base of triangle, *b* (A1)

*e.g.* cos *θ* = , 2 cos *θ*



attempt to substitute valid values into a formula for the area  
of the window (M1)

*e.g.* two triangles plus rectangle, trapezium area formula

correct expression (must be in terms of *θ*) A1

*e.g.*



attempt to replace 2sinθ cosθ by sin 2θ M1

*e.g.* 4 sin *θ* + 2(2 sin *θ* cos *θ*)

*y* = 4 sin *θ* + 2 sin 2*θ* AG N05

(b) correct equation A1

*e.g.* *y* = 5, 4 sin *θ* + 2 sin 2*θ* = 5

evidence of attempt to solve (M1)

*e.g.* a sketch, 4 sin *θ* + 2 sin *θ* – 5 = 0

*θ* = 0.856 (49.0º), *θ* = 1.25 (71.4º) A1A1 N34

(c) recognition that lower area value occurs at *θ* = (M1)



finding value of area at *θ* = (M1)



*e.g.* 4 sin , draw square



*A* = 4 (A1)

recognition that maximum value of *y* is needed (M1)

*A* = 5.19615… (A1)

4 < *A* < 5.20 (accept 4 < *A* < 5.19) A2 N5 7

[16]

**5.** (a) (i) evidence of finding the amplitude (M1)  
*e.g.* , amplitude = 5  
*p* = –5 A1 N2



(ii) period = 8 (A1)  
*q* = 0.785 A1 N2



(iii) *r* = (A1)  
*r* = 2 A1 N2



(b) *k* = –3(accept *y* = –3) A1 N1

[7]

**6.** (a) correct substitution A1*e.g.* 25 + 16 – 40cos *x*, 52 + 42 – 2 × 4 × 5 cos*x*AC = AG



(b) correct substitution A1*e.g.* = 4 sin *x*  
AC = 8 sin *x* A1 N1



(c) (i) evidence of appropriate approach using AC M1*e.g.* 8 sin *x* = , sketch showing intersection



correct solution 8.682..., 111.317... (A1)obtuse value 111.317... (A1)

*x* = 111.32 to 2 dp (do **not** accept the radian answer 1.94) A1 N2

(ii) substituting value of *x* into either expression for AC (M1)  
*e.g.* AC = 8 sin 111.32  
AC = 7.45 A1 N2

(d) (i) evidence of choosing cosine rule (M1)  
*e.g.* cos *B* =



correct substitution A1*e.g.* , 7.452 = 32 – 32 cos *y*, cos *y* = –0.734...  
*y* = 137 A1 N2



(ii) correct substitution into area formula (A1)  
*e.g.* × 4 × 4 × sin 137, 8 sin 137  
area = 5.42 A1 N2



[14]

**7.** (a) appropriate approach (M1)  
*e.g.* 6 = 8*θ*  
 = 0.75 A1 N2



(b) evidence of substitution into formula for area of triangle (M1)  
*e.g.* area = × 8 × 8 × sin(0.75)  
area = 21.8… (A1)



evidence of substitution into formula for area of sector (M1)  
*e.g.* area = × 64 × 0.75  
area of sector = 24 (A1)



evidence of substituting areas (M1)  
*e.g.* , area of sector – area of triangle  
area of shaded region = 2.19 cm2 A1 N4



(c) attempt to set up an equation for area of sector (M1)*e.g.* 45 = × 82 × *θ*  
 = 1.40625 (1.41 to 3 sf) A1 N2



(d) **METHOD 1**

attempting to find angle EOF (M1)  
*e.g.* π – 0.75 – 1.41  
 = 0.985 (seen anywhere) A1



evidence of choosing cosine rule (M1)  
correct substitution A1  
*e.g.* EF =   
EF = 7.57 cm A1 N3



**METHOD 2**

attempting to find angles that are needed (M1)  
*e.g.* angle EOF and angle OEF  
 = 0.9853... **and**  = 1.078... A1



evidence of choosing sine rule (M1)  
correct substitution (A1)  
*e.g.*   
EF = 7.57 cm A1 N3



**METHOD 3**

attempting to find angle EOF (M1)  
*e.g.* π – 0.75 – 1.41  
 = 0.985 (seen anywhere) A1



evidence of using half of triangle EOF (M1)  
*e.g. x* = 8 sin



correct calculation A1  
*e.g.* *x* = 3.78

EF = 7.57 cm A1 N3

[15]

**8.** (a) correct substitution into the formula for the area of a triangle A1*e.g*. × 5 × 13.6 × sin *C* = 20, × 5 × *h* = 20



attempt to solve (M1)  
*e.g.* sin *C* = 0.5882... , sin *C* =



= 36.031...° (0.6288… radians) (A1)  
 = 144° (2.51 radians) A1 N3



(b) evidence of choosing the cosine rule (M1)  
correct substitution A1  
*e.g.* (AB)2 = 52 + 13.62 – 2(5)(13.6)cos143.968...

AB = 17.9 A1 N2

[7]

**9.** (a) valid approach (M1)  
*e.g.* 15 mins is half way, top of the wheel, *d* + 1  
height = 101 (metres) A1 N2

(b) evidence of identifying rotation angle after 6 minutes A1*e.g.* of a rotation, 72°



evidence of appropriate approach (M1)  
*e.g.* drawing a right triangle and using cosine ratio

correct working (seen anywhere) A1  
*e.g.* cos, 15.4(508...)



evidence of appropriate method M1  
*e.g.* height = radius + 1 – 15.45...

height 35.5 (metres) (accept 35.6) A1 N2

(c) **METHOD 1**

evidence of substituting into *b* = (M1)



correct substitution  
*e.g.* period = 30 minutes, *b* = A1*b* = 0.209 A1 N2



substituting into *h*(*t*) (M1)  
*e.g. h*(0) = 1, *h*(15) = 101

correct substitution A1  
1 = 50 sin + 51



*c* = 7.5 A1 N2

**METHOD 2**

evidence of setting up a system of equations (M1)

two correct equations  
*e.g.* 1 = 50 sin *b*(0 – *c*) + 51, 101 = 50 sin *b*(15 – *c*) + 51 A1A1

attempt to solve simultaneously (M1)  
*e.g.* evidence of combining two equations  
*b* = 0.209 , *c* = 7.5 A1A1 N2N2



(d) evidence of solving *h*(*t*) = 96 (M1)  
*e.g.* equation, graph

*t* = 12.8 (minutes) A2 N3

[16]

**10.** (a) **METHOD 1**

choosing cosine rule (M1)  
substituting correctly A1  
*e.g.* AB =   
AB = 6.11(cm) A1 N2



**METHOD 2**

evidence of approach involving right-angled triangles (M1)  
substituting correctly A1  
*e.g.* sin 0.9 = AB = 3.9 sin 0.9  
AB = 6.11 (cm) A1 N2



**METHOD 3**

choosing the sine rule (M1)  
substituting correctly A1  
*e.g.*   
AB = 6.11 (cm) A1 N2



(b) **METHOD 1**

reflex = 2π – 1.8 (= 4.4832) (A2)  
correct substitution *A* = (3.9)2(4.4832...) A1  
area = 34.1 (cm2) A1 N2



**METHOD 2**

finding area of circle *A* = π(3.9)2 (= 47.78...) (A1)  
finding area of (minor) sector *A* = (3.9)2(1.8) (= 13.68...) (A1)  
subtracting M1  
*e.g.* π(3.9)2 – 0.5(3.9)2(1.8), 47.8 – 13.7  
area = 34.1 (cm2) A1 N2



**METHOD 3**finding reflex = 2π – 1.8 (= 4.4832) (A2)  
finding proportion of total area of circle A1  
*e.g.*   
area = 34.1 (cm) A1 N2



[7]

**11.** (a) attempt to form any composition (even if order is reversed) (M1)correct composition *h*(*x*) = (A1)  
 A1 N3



(b) period is 4π(12.6) A1 N1

(c) range is –5 ≤ *h*(*x*) ≤ 3 ([–5, 3]) A1A1 N2

[6]

**12.** (a) **METHOD 1**

evidence of choosing the cosine formula (M1)  
correct substitution A1  
*e.g.*   
 = 2.38 radians (= 136°) A1 N2



**METHOD 2**

evidence of **appropriate** approach involving right-angled triangles (M1)correct substitution A1*e.g.*   
 = 2.38 radians (= 136°) A1 N2



(b) **METHOD 1**

= π – 2.381 (180 – 136.4) (A1)evidence of choosing the sine rule in triangle ACD (M1)correct substitution A1  
*e.g.*   
 = 0.836... (= 47.9...°) A1  
 = π – (0.760... + 0.836...) (180 – (43.5... + 47.9...))  
= 1.54 (= 88.5°) A1 N3



**METHOD 2**

(A1)evidence of choosing the sine rule in triangle ABD (M1)correct substitution A1*e.g.*



= 0.836... (= 47.9...°) A1 = π – 0.836... – (π – 2.381...) (= 180 – 47.9... – (180 – 136.4))  
= 1.54 (= 88.5°) A1 N3



**Note:** Two triangles are possible with the given information.  
 If candidate finds = 2.31 (132°) leading to  
 = 0.076 (4.35°), award marks as  
 per markscheme.



[8]

**13.** (a) choosing sine rule (M1)  
correct substitution A1  
*e.g.*



AD = 9.71 (cm) A1 N2

(b) **METHOD 1**

finding angle OAD = π – 1.1 = (2.04) (seen anywhere) (A1)

choosing cosine rule (M1)  
correct substitution A1  
*e.g.* OD2 = 9.712 + 42 – 2 × 9.71 × 4 × cos(π – 1.1)

OD = 12.1 (cm) A1 N3

**METHOD 2**

finding angle OAD = π – 1.1 = (2.04) (seen anywhere) (A1)

choosing sine rule (M1)  
correct substitution A1

*e.g.*



OD = 12.1 (cm) A1 N3

(c) correct substitution into area of a sector formula (A1)*e.g.* area = 0.5 × 42 × 0.8  
 area = 6.4 (cm2) A1 N2

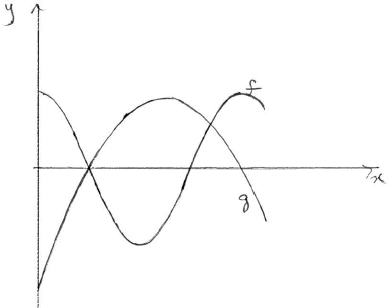
(d) substitution into area of triangle formula OAD (M1)  
correct substitution A1  
*e.g. A* = × 4 × 12.1 × sin 0.8, *A* = × 4 × 9.71 × sin 2.04,  
*A* = × 12.1 × 9.71 × sin 0.3



subtracting area of sector OABC from area of triangle OAD (M1)  
*e.g.* area ABCD = 17.3067 – 6.4  
 area ABCD = 10.9 (cm2) A1 N2

[13]

**14.** (a)  
 A1A1A1 N3



**Note:** Award A1 for f being of sinusoidal shape, with  
 2 maxima and one minimum,  
 A1 for g being a parabola opening down,  
 A1 for **two** intersection points in approximately  
 correct position.

(b) (i) (2,0) (accept *x* = 2) A1 N1

(ii) period = 8 A2 N2

(iii) amplitude = 5 A1 N1

(c) (i) (2, 0), (8, 0) (accept *x* = 2, *x* = 8) A1A1 N1N1

(ii) *x* = 5 (must be an equation) A1 N1

(d) **METHOD 1**

intersect when *x* = 2and *x* = 6.79(may be seen as limits of integration) A1A1

evidence of approach (M1)*e.g.*

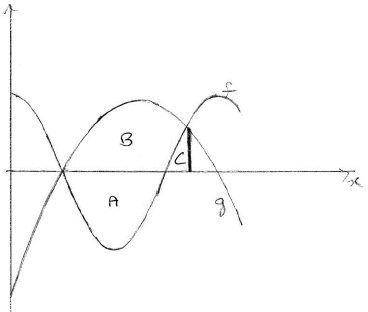


area = 27.6 A2 N3

**METHOD 2**

intersect when *x* = 2and *x* = 6.79(seen anywhere) A1A1

evidence of approach using a sketch of *g* and *f*, or *g* – *f*. (M1)



*e.g.* area *A* + *B* – *C*, 12.7324 + 16.0938 – 1.18129...  
area = 27.6 A2 N3

[15]

**15.** (a) choosing sine rule (M1)

correct substitution A1



sin *R* = 0.676148...

= 42.5 A1 N2



(b) *P* = 180  75  *R*

*P* = 62.5 (A1)

substitution into any correct formula A1

*e.g.* area  PQR = (their *P*)



= 31.0 (cm2) A1 N2

[6]

**16.** (a) evidence of appropriate approach M1

*e.g.* 3 =



*r* =13.5 (cm) A1 N1

(b) adding two radii plus 3 (M1)

perimeter = 27+3 (cm) (= 36.4) A1 N2

(c) evidence of appropriate approach M1

*e.g.*



area = 20.25 (cm2) (= 63.6) A1 N1

[6]

**17.** (a)



A1A1A1 N3

**Note:** Award A1 for passing through (0, 0), A1  
 for correct shape, A1 for a range of  
 approximately 1 to 15.

(b) evidence of attempt to solve *f* (*x*) = 1 (M1)

*e.g.* line on sketch, using



*x* = 0.207 *x* = 0.772 A1A1 N3

[6]

**18.** (a) (i) 7 A1 N1

(ii) 1 A1 N1

(iii) 10 A1 N1

(b) (i) evidence of appropriate approach M1

*e.g.*



*A* = 8 AG N0

(ii) *C* = 10 A2 N2

(iii) **METHOD 1**

period = 12 (A1)

evidence of using *B*  period = 2 (accept 360) (M1)

*e.g.* 12 =



(accept 0.524 or 30) A1 N3



**METHOD 2**

evidence of substituting (M1)

*e.g.* 10 = 8 cos 3*B* + 10

simplifying (A1)

*e.g.* cos 3*B* = 0



(accept 0.524 or 30) A1 N3



(c) correct answers A1A1

*e.g.* *t* = 3.52, *t* = 10.5, between 03:31 and 10:29 (accept 10:30) N2

[11]

**19.** (a) finding = 110° (= 1.92 radians) (A1)evidence of choosing cosine rule (M1)*e.g.* AC2 = AB2 + BC2 – 2(AB)(BC) cos   
correct substitution A1*e.g.* AC2 = 252 + 402 – 2(25)(40) cos 110°  
AC = 53.9 (km) A1 N3



(b) **METHOD 1**

correct substitution into the sine rule A1*e.g.*   
 = 44.2° A1bearing = 074° A1 N1



**METHOD 2**

correct substitution into the cosine rule A1*e.g.*   
 = 44.3° A1bearing = 074° A1 N1



[7]

**20.** (a) using the cosine rule *a*2= *b*2+ *c*2– 2*bc* (M1)



substituting correctly BC2 = 652 + 1042 – 2(65)(104)cos60° A1  
= 4225 + 10 816 – 6760 = 8281  
 BC = 91m A1 N2



(b) finding the area, using (M1)substituting correctly, area = (65)(104)sin60° A1= (accept *p* = 1690) A1 N2



(c) (i) *A*1 = (65)(*x*)sin30° A1  
= AG N0



(ii) *A*2 = (104)(*x*)sin30° M1= 26*x* A1 N1



(iii) stating *A*1+ *A*2= *A* or substituting + 26*x* = (M1)  
simplifying A1*x* = A1  
 *x* = (accept *q* = 40) A1 N2



(d) (i) Recognizing that supplementary angles have equal sines  
*e.g.* = 180° – R1



(ii) using sin rule in ∆ADB and ∆ACD (M1)  
substituting correctly A1and M1  
since   
 A1  
 AG N0



[18]

**21.** ***Notes****: Candidates may have differing answers due to using approximate answers from previous parts or using answers from the GDC.   
Some leeway is provided to accommodate this.*

(a) **METHOD 1**

Evidence of using the cosine rule (M1)

*eg* cos ***C*** =



Correct substitution

*eg* cos = A1



cos = 0.25



= 1.82 (radians) A1 N2



**METHOD 2**

Area of AOBP = 5.81 (from part (d))

Area of triangle AOP = 2.905 (M1)

2.9050 = 0.5  2  3  sin A1



= 1.32 or 1.82



= 1.82 (radians) A1 N2



(b) = 2(  1.82) (= 2  3.64) (A1)



= 2.64 (radians) A1 N2



(c) (i) Appropriate method of finding area (M1)

*eg* area =



Area of sector PAEB = A1



= 13.0 (cm2)  
(accept the exact value 13.04) A1 N2

(ii) Area of sector OADB = A1



= 11.9 (cm2) A1 N1

(d) (i) Area AOBE = Area PAEB  Area AOBP (= 13.0  5.81) M1

= 7.19 (accept 7.23 from the exact answer for PAEB) A1 N1

(ii) Area shaded = Area OADB  Area AOBE (= 11.9  7.19) M1

= 4.71 (accept answers between 4.63 and 4.72) A1 N1

[14]

**22.** (a) Evidence of choosing cosine rule (M1)

*eg* *a*2 = *b*2 + *c*2  2*bc* cos *A*

Correct substitution A1

*eg* (AD)2 = 7.12 + 9.22  2(7.1) (9.2) cos 60

(AD)2 = 69.73 (A1)

AD = 8.35 (cm) A1 N2

(b) 180  162 = 18 (A1)

Evidence of choosing sine rule (M1)

Correct substitution A1

*eg* =



DE = 2.75 (cm) A1 N2

(c) Setting up equation (M1)

*eg* *ab* sin *C* = 5.68, *bh* = 5.68



Correct substitution A1

*eg* 5.68 = (3.2) (7.1) sin ,  3.2  *h* = 5.68, (*h* = 3.55)



sin = 0.5 (A1)



30 and/or 150 A1 N2



(d) Finding AC (60 + DC) (A1)



Using appropriate formula (M1)

*eg* (AC)2 = (AB)2 + (BC)2, (AC)2 = (AB)2 + (BC)2  2 (AB)  
(BC) cos ABC

Correct substitution (allow ***FT*** on **their** seen )



*eg* (AC)2 = 9.22 + 3.22 A1

AC = 9.74 (cm) A1 N3

(e) For finding area of triangle ABD (M1)

Correct substitution Area =  9.2  7.1 sin 60 A1



= 28.28... A1

Area of ABCD = 28.28... + 5.68 (M1)

= 34.0 (cm2) A1 N3

[21]

**23.** (a) **METHOD 1**

Using the discriminant  = 0 (M1)

*k*2 = 4  4  1

*k* = 4, *k* =  4 A1A1 N3

**METHOD 2**

Factorizing (M1)

(2*x*  1)2

*k* = 4, *k* =  4 A1A1 N3

(b) Evidence of using cos 2** = 2 cos2 **  1 M1

*eg* 2(2 cos2 **  1) + 4 cos ** + 3

*f* (**) = 4 cos2 ** + 4 cos ** + 1 AG N0

(c) (i) 1 A1 N1

(ii) **METHOD 1**

Attempting to solve for cos ** M1

cos ** = (A1)



** = 240, 120,  240, 120 (correct four values only) A2 N3

**METHOD 2**

Sketch of *y* = 4 cos2 ** + 4 cos ** + 1 M1



Indicating 4 zeros (A1)

** = 240, 120, 240, 120 (correct four values only) A2 N3

(d) Using sketch (M1)

*c* = 9 A1 N2

[11]

**24.** ***Note:*** *Accept* ***exact*** *answers given in terms of .*

(a) Evidence of using *l* = ***r***** (M1)

arc AB = 7.85 (m) A1 N2

(b) Evidence of using (M1)



Area of sector AOB = 58.9 (m2) A1 N2

(c) **METHOD 1**



angle = (A1)



attempt to find 15 sin M1



height = 15 + 15 sin



= 22.5 (m) A1 N2

**METHOD 2**



angle = (A1)



attempt to find 15 cos M1



height = 15 + 15 cos



= 22.5 (m) A1 N2

(d) (i) (M1)



= 25.6 (m) A1 N2

(ii) *h*(0) = 15  15 cos (M1)



= 4.39(m) A1 N2

(iii) **METHOD 1**

Highest point when *h* = 30 R1

30 = 15  15 cos M1



cos = 1 (A1)



*t* = 1.18 A1 N2



**METHOD 2**



Sketch of graph of *h* M2

Correct maximum indicated (A1)

*t* = 1.18 A1 N2

**METHOD 3**

Evidence of setting *h*(*t*) = 0 M1

sin (A1)



Justification of maximum R1

*eg* reasoning from diagram, first derivative test, second  
derivative test

*t* = 1.18 A1 N2



(e) *h*(*t*) = 30 sin (may be seen in part (d)) A1A1 N2



(f) (i)



A1A1A1 N3

**Notes:** Award A1 for range 30 to 30, A1  
 for two zeros.

Award A1 for approximate correct  
 **sinusoidal** shape.

(ii) **METHOD 1**

Maximum on graph of *h* (M1)

*t* = 0.393 A1 N2

**METHOD 2**

Minimum on graph of *h* (M1)

*t* = 1.96 A1 N2

**METHOD 3**

Solving *h*(t) = 0 (M1)

One or both correct answers A1

*t* = 0.393, *t* = 1.96 N2

[22]

**25.** (a) For **correct** substitution into cosine rule A1

BD =



For factorizing 16, BD = A1



= AG N0



(b) (i) BD = 5.5653 ... (A1)

M1A1



sin = 0.911 (accept 0.910, subject to ***AP***) A1 N2



(ii) = 65.7 A1 N1



Or = 180  their acute angle (M1)



= 114 A1 N2

(iii) = 89.3 (A1)



(or cosine rule) M1A1



BC = 13.2 (accept 13.17…) A1

Perimeter = 4 + 8 + 12 + 13.2

= 37.2 A1 N2

(c) Area =  4  8  sin 40 A1



= 10.3 A1 N1

[16]

**26.** (a) (i) OP = PQ (= 3cm) R1

So  OPQ is isosceles AG N0

(ii) Using cos rule correctly *eg* cos = (M1)



cos = A1



cos = AG N0



(iii) Evidence of using sin2 *A* + cos2 *A* = 1 M1

sin = A1



sin = AG N0



(iv) Evidence of using area triangle OPQ = M1



*eg*



Area triangle OPQ = A1 N1



(b) (i) = 1.4594...



= 1.46 A1 N1



(ii) Evidence of using formula for area of a sector (M1)

*eg* Area sector OPQ =



= 6.57 A1 N2

(c) = (A1)



Area sector QOS = A1



= 6.73 A1 N2

(d) Area of small semi-circle is 4.5 (= 14.137...) A1

Evidence of correct approach M1

*eg* Area = area of semi-circle  area sector OPQ  area sector QOS +  
area triangle POQ

Correct expression A1

*eg* 4.5  6.5675...  6.7285... + 4.472..., 4.5  (6.7285... + 2.095...),

4.5 (6.5675... + 2.256...)

Area of the shaded region = 5.31 A1 N1

[17]

**27.** (a) using the cosine rule *(A2)* = *b*2 + *c*2 –2*bc* cos (M1)  
substituting correctly BC2 = 652 +1042 –2 (65) (104) cos 60° A1  
= 4225 + 10816 – 6760 = 8281  
 BC = 91 m A1 3



(b) finding the area, using *bc* sin (M1)  
substituting correctly, area = (65) (104) sin 60° A1  
= 1690 (Accept *p* = 1690) A1 3



(c) (i) *A*1 = (65) (*x*) sin 30° A1  
= AG 1



(ii) *A*2 = (104) (*x*) sin 30° M1  
= 26*x* A1 2



(iii) starting *A*1 + *A*2 = *A* or substituting + 26*x* = 1690 (M1)  
simplifying = 1690 A1



*x* = A1



* x* = 40 (Accept *q* = 40) A1 4



(d) (i) Recognizing that supplementary angles have equal sines   
eg = 180 –  sin = sin R1



(ii) using sin rule in ΔADB and ΔACD (M1)

substituting correctly A1



and M1



since sin = sin



A1



 AG 5



[18]

**28.** (a) for using cosine rule (M1)



(A1)



(A1) (N0) 3



**Notes:** Either the first or the second line may be implied, but not both. Award **no marks** if 8.24 is obtained by assuming a right (angled) triangle (BC = 17 sin 29).

(i)



for using sine rule (may be implied) (M1)



(A1)



(A1) (N2)



(ii) (A1)



(Accept) (A1) (N1) 5



(c) from previous triangle



Therefore alternative (A1)



(M1)(A1)



(A1) (N1) 4



(d)



Minimum length for BC when = 90°or diagram  
showing right triangle (M1)



(A1) (N1) 2



[14]

**29.** (a) (i)



(A1)(A1) (N2)



**Note:** Award (A1)(A1) for only if work shown, using product rule on .



(ii) or  
 (A1) (N1)



(iii)



(A1)(A1)(A1) (N1)  
 (N1)(N1) 6



(b) (A1) (N1) 1



(c) (i) **EITHER**

curve crosses axis when (may be implied) (A1)



(M1)(A1) (N3)



**OR**

Area = (M1)(A2) (N3)



(ii) Area (M1)



(A1) (N2) 5



[12]

**30.** (a) (i) 10 + 4 sin 1 = 13.4 (A1)

(ii) At 2100, *t* = 21 (A1)  
10 + 4 sin 10.5 = 6.48 (A1) (N2) 3

**Note:** Award (A0)(A1) if candidates use t = 2100 leading to y = 12.6. No other **ft** allowed.

(b) (i) 14 metres (A1)

(ii) 14 = 10 + 4 sin  sin = 1 (M1)  
  *t* = π (3.14) (correct answer only) (A1) (N2) 3



(c) (i) 4 (A1)

(ii) 10 + 4 sin = 7 (M1)  
 sin = –0.75 (A1)  
 *t* = 7.98 (A1) (N3)



(iii) depth < 7 from 8 –11 = 3 hours (M1)  
from 2030 – 2330 = 3 hours (M1)  
therefore, total = 6 hours (A1) (N3)7

[13]

**31.** (a) (i) (A1)(A1)



(ii) **EITHER**

(M1)



**OR**

,  
for (M1)



**THEN**

(A1)(A1)(A1) (N4) 6



(b) (i) translation (A1)

in the *y*-direction of –1 (A1)

(ii) 1.11 (1.10 from TRACE is subject to **AP**) (A2) 4

[10]

**32.** (a)   
 = ***q* *–* *p* =**  (A1)(A1)  
 = (A1) 3



(b) cos (A1)  
= , = (A1)(A1)  
 = –21 + 6 = –15 (A1)  
cos (AG) 4



(c) (i) Since + = 180° (R1)  
cos = –cos (AG)



(ii) sin = (M1)  
 = (A1)  
 = (AG)  
**OR**cos ** *=*



(M1)

therefore *x*2 = 754 – 225 = 529  *x* = 23 (A1)  
 sin ** *=* (AG)



**Note:** Award (A1)(A0) for the following solution.

cos ** *=*  ** *=* 56.89°  
  sin ** *=* 0.8376  
 *=* 0.8376  sin ** *=*



(iii) Area of OPQR = 2 (area of triangle PQR) (M1)  
 = 2 × (A1)  
 = 2 × (A1)  
 = 23 sq units. (A1)  
**OR**Area of OPQR = 2 (area of triangle OPQ) (M1)  
 = 2 (A1)(A1)  
 = 23 sq units. (A1) 7



**Notes:** Other valid methods can be used.  
Award final (A1) for the **integer** answer.

[14]

**33.** (a) Sine rule (M1)(A1)  
 PR =   
 = 5.96 km (A1) 3



(b) **EITHER**

Sine rule to find PQ  
 PQ = (M1)(A1)  
 = 4.39 km (A1)



**OR**

Cosine rule: PQ2 = 5.962 + 92 – (2)(5.96)(9) cos 25 (M1)(A1)  
 = 19.29  
 PQ = 4.39 km (A1)

Time for Tom = (A1)



Time for Alan = (A1)  
 Then = (M1)  
 *a* = 10.9 (A1) 7



(c) RS2 = 4QS2 (A1)  
4QS2 = QS2 + 81 – 18 × QS × cos 35 (M1)(A1)  
3QS2 + 14.74QS – 81 = 0 (**or** 3*x*2 + 14.74*x* *–* 81 = 0) (A1)  
 QS = –8.20 **or** QS = 3.29 (G1)  
therefore QS = 3.29 (A1)  
**OR** (M1)  
 sin sin 35 (A1)  
 = 16.7° (A1)  
Therefore, = 180 – (35 + 16.7)  
 = 128.3° (A1)  
 (M1)  
 QS =   
 = 3.29 (A1) 6



[16]

**34.** (a) (i) cos , sin (A1)  
therefore cos = 0 (AG)



(ii) cos *x* + sin *x* = 0  1 + tan *x* = 0  
 tan *x* = –l (M1)  
*x* = (A1)



**Note:** Award (A0) for 2.36.

**OR***x* = (G2) 3



(b) *y* = e*x*(cos *x* + sin *x*)  
 = e*x*(cos *x* + sin *x*) + e*x*(–sin *x* + cos *x*) (M1)(A1)(A1) 3  
 = 2e*x* cos *x*



(c) = 0 for a turning point  2e*x* cos *x* = 0 (M1)  
 cos *x* = 0 (A1)  
 *x* *=*  *a* *=* (A1)  
*y* *=* e(cos *+* sin ) = e  
*b* *=* e (A1) 4



**Note:** Award (M1)(A1)(A0)(A0) for a = 1.57, b = 4.81.

(d) At D, = 0 (M1)  
2e*x* cos *x* – 2e*x*sin *x* = 0 (A1)  
2e*x* (cos *x* – sin *x*) = 0  
 cos *x* – sin *x* = 0 (A1)  
 *x* = (A1)  
 *y* = e(cos *+* sin ) (A1)  
 = e (AG) 5



(e) Required area = (cos *x* + sin *x*)d*x* (M1)  
 = 7.46 sq units (G1)  
**OR**Αrea = 7.46 sq units (G2) 2



**Note**: Award (M1)(G0) for the answer 9.81 obtained if the calculator is in degree mode.

[17]

**35.** ***Note:*** *Do* *not* *penalize* *missing* *units* *in* *this* *question.*

(a) AB2 = 122 + 122 – 2 × 12 × 12 × cos 75° (A1)  
 = 122(2 – 2 cos 75°) (A1)  
 = 122 × 2(1 cos 75°)  
AB = 12 (AG) 2



**Note:** The second (A1) is for transforming the initial expression to any simplified expression from which the given result can be clearly seen.

(b) = 37.5° (A1)  
BP = 12 tan 37.5° (M1)  
 = 9.21 cm (A1)



**OR**

= 105° = 37.5° (A1)  
 (M1)  
BP = = 9.21(cm) (A1) 3



(c) (i) Area ∆OBP = (M1)  
 = 55.3 (cm2) (accept 55.2 cm2) (A1)



(ii) Area ∆ABP = (9.21)2 sin105° (M1)  
 = 41.0 (cm2) (accept 40.9 cm2) (A1) 4



(d) Area of sector = (M1)  
 = 94.2 (cm2) (accept 30π or 94.3 (cm2)) (A1) 2



(e) Shaded area = 2 × area ∆OPB – area sector (M1)  
 = 16.4 (cm2) (accept 16.2 cm2, 16.3 cm2) (A1) 2

[13]

**36.** ***Note****:* *Do* *not* *penalize* *missing* *units* *in* *this* *question.*

(a) (i) At release(P), *t* = 0 (M1)  
 *s* = 48 + 10 cos 0  
 = 58 cm below ceiling (A1)

(ii) 58 = 48 +10 cos 2π*t* (M1)  
cos 2π*t* = 1 (A1)  
*t* = 1sec (A1)

**OR**

*t* = 1sec (G3) 5

(b) (i) = –20π sin 2π*t* (A1)(A1)



**Note:** Award (A1) for –20π, and (A1) for sin 2t.

(ii) *v* = = –20π sin 2π*t* = 0 (M1)  
sin 2 π*t* = 0  
*t* = 0, ... (at least 2 values) (A1)  
*s* = 48 + 10 cos 0 **or** *s* = 48 +10 cos π (M1)  
 = 58 cm (at P) = 38 cm (20 cm above P) (A1)(A1) 7



**Note:** Accept these answers without working for full marks. May be deduced from recognizing that amplitude is 10.

(c) 48 +10 cos 2π*t* = 60 + 15 cos 4π*t* (M1)*t* = 0.162 secs (A1)

**OR**

*t* = 0.162 secs (G2) 2

(d) 12 times (G2) 2

**Note:** If either of the correct answers to parts (c) and (d) are missing and suitable graphs have been sketched, award (G2) for sketch of suitable graph(s); (A1) for t = 0.162; (A1) for 12.

[16]

**37.** (a) (M1)  
sin A = (A1)  
 = (AG) 2



(b)



(i) + = 180° (A1)



(ii) sin A =   
=> A = 59.0° or 121° (3 sf) (A1)(A1)  
=> = 180° – (121° + 45°)  
 = 14.0° (3 sf) (A1)



(iii) (M1)  
=>BD = 1.69 (A1) 6



(c) (M1)(A1)  
 = (AG) 2



**OR**

(M1)(A1)



= (AG) 2



[10]

**38.** (a) (i) AP = (M1) (AG)



(ii) OP = (A1) 2



(b) (M1)  
= (M1)  
= (M1)  
 (AG) 3



(c) For *x* = 8, = 0.780869 (M1)  
arccos 0.780869 = 38.7° (3 sf) (A1)



**OR**

(M1)  
 = arctan (0.8) = 38.7° (3 sf) (A1) 2



(d) = 60°  = 0.5  
0.5 = (M1)  
2*x*2 – 16*x* + 80 – = 0 (M1)  
*x* = 5.63 (G2) 4



(e) (i) *f* (*x*) = 1 when = 1 (R1)  
hence, when = 0. (R1)  
This occurs when the points O, A, P are collinear. (R1)



(ii) The line (OA) has equation *y* = (M1)  
When *y* = 10, *x* = (= 13) (A1)



**OR**

*x* = (= 13) (G2) 5



**Note:** Award (G1) for 13.3.

[16]

**39.** (a)  
 5



(b)  is a solution if and only if  +  cos  = 0. (M1)  
Now  +  cos  =  + (–1) (A1)  
 = 0 (A1) 3

(c) By using appropriate calculator functions *x* = 3.696 722 9... (M1)  
 *x* = 3.69672 (6sf) (A1) 2

(d) See graph: (A1)  
 (A1) 2



(e) **EITHER** = 7.86960 (6 sf) (A3) 3



**Note:** This answer assumes appropriate use of a calculator eg  
 ‘fnInt’:



**OR**   
= ( – 0) + ( sin  – 0 × sin 0) + (cos  – cos 0) (A1)  
= 2 + 0 + –2 = 7.86960 (6 sf) (A1) 3



[15]

**40.** (a) (i) *Q* = (14.6 – 8.2) (M1)  
= 3.2 (A1)



(ii) *P* = (14.6 + 8.2) (M0)  
= 11.4 (A1) 3



(b) 10 = 11.4 + 3.2 cos (M1)  
so = cos   
therefore arccos (A1)  
which gives 2.0236... = *t* or *t* = 3.8648. *t* = 3.86(3 sf) (A1) 3



(c) (i) By symmetry, ne*x*t time is 12 – 3.86... = 8.135... *t* = 8.14 (3 sf) (A1)

(ii) From above, first interval is 3.86 < *t* < 8.14 (A1)

This will happen again, 12 hours later, so (M1)  
15.9 < *t* < 20.1 (A1) 4

[10]

**41.** (a) *y* =  sin *x* – *x*  
 (A5) 5



**Notes:** Award (A1) for appropriate scales marked on the axes.  
Award (A1) for the x-intercepts at (2.3, 0).  
Award (A1) for the maximum and minimum points at (1.25, 1.73).  
Award (A1) for the end points at (3, 2.55).  
Award (A1) for a smooth curve.  
Allow some flexibility, especially in the middle three marks here.

(b) *x* = 2.31 (A1) 1

(c) (A1)(A1)



**Note:** Do not penalize for the absence of C.

Required area = (M1)  
 = 0.944 (G1)



**OR** area = 0.944 (G2) 4

[10]

**42.** (a)(i) & (c)(i)

(A3)



**Notes:** The sketch does **not** need to be on graph paper. It should have the correct shape, and the points (0, 0), (1.1, 0.55), (1.57, 0) and (2, –1.66) should be indicated in some way.  
Award (A1) for the correct shape.  
Award (A2) for 3 or 4 correctly indicated points, (A1) for 1 or 2 points.

(ii) Approximate positions are  
positive *x*-intercept (1.57, 0) (A1)  
maximum point (1.1, 0.55) (A1)  
end points (0, 0) and (2, –1.66) (A1)(A1) 7

(b) *x*2 cos *x* = 0 *x* ≠ 0 ⇒ cos *x* = 0 (M1)  
  *x* = (A1) 2



**Note:** Award (A2) if answer correct.

(c) (i) see graph (A1)

(ii) cos *x* d*x* (A2) 3



**Note:** Award (A1) for limits, (A1) for rest of integral correct (do not penalize missing dx).

(d) Integral = 0.467 (G3)

**OR**

Integral = (M1)  
= – [0 + 0 – 0] (M1)  
= – 2 (exact) **or** 0.467 (3 sf) (A1) 3



[15]

**43.** (a) From graph, period = 2π (A1) 1

(b) Range = {*y* –0.4 < *y* < 0.4} (A1) 1

(c) (i) *f* (*x*) = {cos *x* (sin *x*)2}  
= cos *x* (2 sin *x* cos *x*) – sin *x* (sin *x*)2 **or** –3 sin3 *x* + 2 sin *x* (M1)(A1)(A1)



**Note:** Award (M1) for using the product rule and (A1) for each part.

(ii) *f* (*x*) = 0 (M1)  
 sin *x*{2 cos *x* – sin2 *x*} = 0 **or** sin *x*{3 cos *x* – 1} = 0 (A1)  
 3 cos2 *x* – 1 = 0  
 cos *x* = ± (A1)  
At A, *f* (*x*) > 0, hence cos *x* = (R1)(AG)



(iii) *f* (*x*) = (M1)  
 = (A1) 9



(d) *x* = (A1) 1



(e) (i) (M1)(A1)



(ii) Area = (M1)  
 = (A1) 4



(f) At C *f* (*x*) = 0 (M1)  
 9 cos3 *x* – 7 cos *x* = 0  
 cos *x*(9 cos2 *x* – 7) = 0 (M1)  
 *x* = (reject) ***or*** *x* = arccos = 0.491 (3 sf)(A1)(A1) 4



[20]

**44.** (a) *f* (1) = 3 *f* (5) = 3 (A1)(A1) 2

(b) **EITHER** distance between successive maxima = period (M1)  
 = 5 – 1 (A1)  
 = 4 (AG)

**OR** Period of sin *kx* = ; (M1)  
 so period = (A1)  
 = 4 (AG) 2



(c) **EITHER** *A* sin + *B* = 3 and *A* sin+ *B* = –1 (M1) (M1)  
 *A* + *B* = 3, – *A* + *B* = –1 (A1)(A1)  
 *A* = 2, *B* = 1 (AG)(A1)  
**OR** Amplitude = *A* (M1)  
 *A* = (M1)  
 *A* = 2 (AG)  
Midpoint value = *B* (M1)  
 *B* = (M1)  
 *B* = 1 (A1) 5



**Note:** As the values of A = 2 and B = 1 are likely to be quite obvious to a bright student, do not insist on too detailed a proof.

(d) *f* (*x*) = 2 sin + 1  
*f* (*x*) = + 0 (M1)(A2)



**Note:** Award (M1) for the chain rule, (A1) for , (A1) for 2 cos.



=  cos (A1) 4



**Notes:** Since the result is given, make sure that reasoning is valid. In particular, the final (A1) is for simplifying the result of the chain rule calculation. If the preceding steps are not valid, this final mark should not be given. Beware of “fudged” results.

(e) (i) *y* = *k* – *x* is a tangent  – =  cos (M1)  
  –1 = cos (A1)  
  *x* =  or 3 or ...  
  *x* = 2 or 6 ... (A1)



Since 0  *x*  5, we take *x* = 2, so the point is (2, 1) (A1)

(ii) Tangent line is: *y* = –(*x* – 2) + 1 (M1)  
 *y* = (2 + 1) – *x*  
 *k* = 2 + 1 (A1) 6

(f) *f* (*x*) = 2  2 sin + 1 = 2 (A1)  
  sin (A1)  
    
 *x* = (A1)(A1)(A1) 5



[24]