**1.** **METHOD 1**

using double-angle identity (seen anywhere) A1

*e.g.* sin 2*x* = 2sin *x* cos *x,* 2cos *x* = 2sin *x* cos *x*

evidence of valid attempt to solve equation (M1)

*e.g.* 0 = 2sin *x* cos *x* – 2cos *x,* 2cos *x* (1– sin *x*) = 0

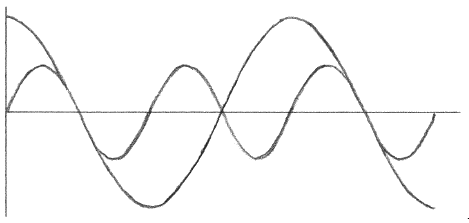
cos *x* = 0, sin *x* =1 A1A1

A1A1A1 N4



[7]

**METHOD 2**



A1A1M1A1

**Notes:** Award A1 for sketch of sin 2x, A1 for a sketch of 2 cos x, M1 for at least one intersection point seen, and A1 for 3 approximately correct intersection points. Accept sketches drawn outside [0, 3π], even those with more than 3 intersections.

A1A1A1 N4



[7]

**2.** (a) (i) 100 (metres) A1N1

(ii) 50 (metres) A1 N1 2

(b) (i) identifying symmetry with *h*(2) = 9.5 (M1)

subtraction A1

*e.g.* 100 – *h*(2), 100 – 9.5

*h*(8) = 90.5 AG N0

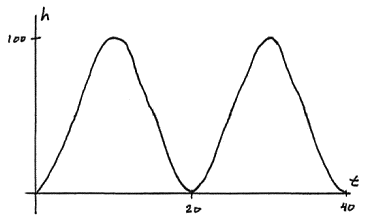
(ii) recognizing period (M1)

*e.g. h*(21) = *h*(1)

*h*(21) = 2.4 A1N2 4

(c)

A1A1A1 N3 3



**Note:** Award A1for end points (0, 0) **and** (40, 0), A1for range 0 ≤ h ≤ 100, A1for approximately correct sinusoidal shape, with two cycles

(d) evidence of a quotient involving 20, 2π or 360º to find *b* (M1)

*e.g.*



(accept *b* =18 if working in degrees) A1 N2

*a* = –50, *c* = 50 A2A1 N35

[14]

**3.** (a) tan *θ* = A1 N1



(b) (i) sin *θ* = , cos *θ* = (A1)(A1)  
correct substitution A1*e.g.* sin 2*θ* = 2  
sin 2*θ* = A1 N3



(ii) correct substitution A1  
*e.g.* cos 2*θ* = 1 – 2  
cos 2*θ* = A1 N1



[7]

**4.** (a) (A1)



= –1 A1 N2

(b) (*g* ° *f*) = *g*(–1) (= 2(–1)2 – 1) (A1)  
= 1 A1 N2



(c) (*g* ° *f*)(*x*) = 2(cos (2*x*))2 – 1 (= 2 cos2(2*x*) – 1) A1

evidence of 2 cos2 *θ* – 1 = cos 2*θ* (seen anywhere) (M1)

(*g* ° *f*)(*x*) = cos 4*x  
k* = 4 A1 N2

[7]

**5.** (a) correct substitution in *l* = *rθ* (A1)  
*e.g.* 10 × × 2π × 10  
arc length = A1 N2



(b) area of large sector = (A1)



area of small sector = (A1)



evidence of valid approach (seen anywhere) M1  
*e.g.* subtracting areas of two sectors,   
area shaded = 6π A1 N3



[6]

**6.** (a) attempt to substitute 1 – 2 sin2 *θ* for cos 2*θ* (M1)correct substitution A1  
*e.g.* 4 – (1 – 2 sin2 *θ*) + 5 sin *θ* 4 – cos 2*θ* + 5 sin *θ* = 2 sin2 *θ* + 5 sin *θ* + 3 AG N0

(b) evidence of appropriate approach to solve (M1)  
*e.g.* factorizing, quadratic formula

correct working A1

*e.g.* (2 sin *θ* + 3)(sin *θ* + 1), (2*x* + 3)(*x* + 1) = 0, sin *x* =



correct solution sin *θ* = –1 (A1)



*θ* = A2 N3



[7]

**7.** (a) (i) *x* = 3 cos *θ* A1 N1

(ii) *y* = 3 sin *θ* A1 N1

(b) finding area (M1)*e.g.* *A* = 2*x* × 2*y*, *A* = 8 × *bh*  
substituting A1  
*e.g.* *A* = 4 × 3 sin *θ* × 3 cos *θ*, 8 × × 3 cos *θ* × 3 sin *θ  
A* = 18(2 sin *θ* cos *θ*) A1  
*A* = 18 sin 2*θ* AG N0



(c) (i) = 36 cos 2*θ* A2 N2



(ii) for setting derivative equal to 0 (M1)  
*e.g.* 36 cos 2*θ* = 0, = 0  
2*θ* = (A1)  
*θ* = A1 N2



(iii) valid reason (seen anywhere) R1  
*e.g.* at ; maximum when *f*″(*x*) < 0



finding second derivative = –72 sin 2*θ* A1  
evidence of substituting M1  
*e.g.* –72 sin  
*θ* = produces the maximum area AG N0



[13]

**8.** (a) (i) evidence of approach (M1)*e.g.* Q – P  
 A1 N2



(ii) A1 N1



(b) **METHOD 1**

choosing correct vectors (A1)(A1)finding (A1) (A1)(A1)  
 = –2 + 4 + 4 (= 6)  
  
substituting into formula for angle between two vectors M1*e.g.* simplifying to expression clearly leading to A1  
*e.g.*   
 AG N0



**METHOD 2**

evidence of choosing cosine rule (seen anywhere) (M1)  
 A1  
 (A1)(A1)(A1)  
 A1  
 A1  
 AG N0



(c) (i) **METHOD 1**

evidence of appropriate approach (M1)  
*e.g.* using , diagram  
substituting correctly (A1)  
*e.g.*   
 A1 N3



**METHOD 2**

since (A1)  
evidence of approach  
*e.g.* drawing a right triangle, finding the missing side (A1) A1 N3



(ii) evidence of appropriate approach (M1)  
*e.g.* attempt to substitute into *ab* sin *C*correct substitution  
*e.g.* area = A1  
area = A1 N2



[16]

**9.** e2*x*(sin *x* + cos *x*) = 0 (A1)  
e2*x* = 0 not possible (seen anywhere) (A1)simplifying  
*e.g.* A1



**EITHER**

tan *x* = A1  
*x* = A2 N4



**OR**

sketch of 30°, 60°, 90° triangle with sides 1, 2, A1  
work leading to *x* = A1  
verifying satisfies equation A1 N4



[6]

**10.** evidence of substituting for cos2*x* (M1)evidence of substituting into sin2 *x* + cos2 *x* = 1 (M1)correct equation in terms of cos *x* (seen anywhere) A1

*e.g.* 2cos2 *x* – 1 – 3 cos *x* – 3 = 1, 2 cos2 *x* – 3 cos *x* – 5 = 0

evidence of appropriate approach to solve (M1)*e.g.* factorizing, quadratic formula

appropriate working A1

*e.g.* (2 cos *x* – 5)(cos *x* + 1) = 0, (2*x* – 5)(*x* + 1), cos *x* =



correct solutions to the equation

*e.g.* cos *x* = , cos *x* = –1, *x* = , *x* = –1 (A1)



*x* = π A1 N4

[7]

**11.** (a) (i) sin 140 = *p* A1 N1

(ii) cos 70 = *q* A1 N1

(b) **METHOD 1**

evidence of using sin2 ** + cos2 ** = 1 (M1)

*e.g.* diagram, (seen anywhere)



cos 140 =  (A1)



cos 140 =  A1 N2



**METHOD 2**

evidence of using cos2 ** = 2 cos2 **  1 (M1)

cos 140 = 2 cos2 70  1 (A1)

cos 140 = 2( *q*)2 1 (= 2*q*2  1) A1 N2

(c) **METHOD 1**

tan 140 = A1 N1



**METHOD 2**

tan 140 = A1 N1



[6]

**12.** (a) period =  A1 N1

(b)



A1A1A1 N3

**Note:** Award A1 for amplitude of 3, A1 for **their**  
 period, A1 for a sine curve passing through  
 (0, 0) and (0, 2).

(c) evidence of appropriate approach (M1)

*e.g.* line *y* = 2 on graph, discussion of number of solutions in  
the domain

4 (solutions) A1 N2

[6]

**13.** (a) evidence of choosing the formula cos2 *A* = 2 cos2 *A*  1 (M1)

**Note:** If they choose another correct formula, do  
 not award the M1 unless there is evidence  
 of finding sin2 A = 1 .



correct substitution A1

*e.g.* cos 2*A* =



A1 N2



(b) **METHOD 1**

evidence of using sin2 *B* + cos2 *B* = 1 (M1)

*e.g.* (seen anywhere),



cos *B* = (A1)



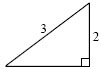
cos *B* = A1 N2



**METHOD 2**

diagram M1

*e.g.*



for finding third side equals (A1)



cos *B* = A1 N2



[6]

**14.** (a) evidence of using area of a triangle (M1)

*e.g.*



*A* = 2 sin ** A1 N2

(b) **METHOD 1**

=   ** (A1)



area OPA = (= 2 sin (  **)) A1



since sin (  **) = sin ** R1

then both triangles have the same area AG N0

**METHOD 2**

triangle OPA has the same height and the same base as triangle OPB R3

then both triangles have the same area AG N0

(c) area semi-circle = A1



area  APB = 2 sin ** + 2 sin ** (= 4 sin **) A1

*S* = area of semicircle  area APB (= 2  4 sin **) M1

*S* = 2( − 2 sin **) AG N0

(d) **METHOD 1**

attempt to differentiate (M1)

*e.g.*



setting derivative equal to 0 (M1)

correct equation A1

*e.g.* 4 cos ** = 0, cos ** = 0, 4 cos ** = 0

** = A1 N3



**EITHER**

evidence of using second derivative (M1)

*S*(**) = 4 sin ** A1

*S* A1



it is a minimum because *S* R1 N0



**OR**

evidence of using first derivative (M1)

for * <*  *S* (**) < 0 (may use diagram) A1



for * >*  *S* (**) > 0 (may use diagram) A1



it is a minimum since the derivative goes from negative  
to positive R1 N0

**METHOD 2**

2  4 sin ** is minimum when 4 sin ** is a maximum R3

4 sin ** is a maximum when sin ** = 1 (A2)

** = A3 N3



(e) *S* is greatest when 4 sin ** is smallest (or equivalent) (R1)

** = 0 (or ) A1 N2

[18]

**15.** (a) changing tan *x* into A1  
*e.g.* sin3 *x* + cos3 *x*



simplifying A1  
*e.g.* sin *x* (sin2 *x* + cos2 *x*), sin3 *x* + sin *x* – sin3 *x*  
*f*(*x*) = sin *x* AG N0

(b) recognizing *f*(2*x*) = sin 2*x*, seen anywhere (A1)evidence of using double angle identity sin (2*x*) = 2 sin *x* cos *x*,  
seen anywhere (M1)  
evidence of using Pythagoras with sin *x =*  M1  
*e.g.* sketch of right triangle, sin2 *x* + cos2 *x* = 1  
cos *x* = (A1)*f*(2*x*) = 2 A1  
*f*(2*x*) = AG N0



[7]

**16.** (a) (i) attempt to substitute (M1)*e.g.* *a* =   
*a* = 7 (accept *a* = –7) A1 N2



(ii) period = 12 (A1)  
*b* = A1 *b* = AG N0



(iii) attempt to substitute (M1)  
*e.g.* *d* =   
*d* = 22 A1 N2



(iv) *c* = 3 (accept *c* = 9 from *a* = –7) A1 N1

**Note:** Other correct values for c can be found,  
 c = 3 ± 12k, k .



(b) stretch takes 3 to 1.5 (A1)

translation maps (1.5, 29) to (4.5, 19) (so M′ is (4.5, 19)) A1 N2

(c) *g*(*t*) = 7 cos(*t* – 4.5) + 12 A1A2A1 N4



**Note:** Award A1for , **A2** for 4.5, A1for 12.  
 Other correct values for c can be found  
 c = 4.5 ± 6k, k .



(d) translation (A1)horizontal stretch of a scale factor of 2 (A1)completely correct description, in correct order A1 N3*e.g.* translation then horizontal stretch of a scale factor of 2



[16]

**17.** (a) *p* = 30 A2 N2

(b) **METHOD 1**Period = (M2)= (A1) *q* = 4 A1 N4



**METHOD 2**Horizontal stretch of scale factor = (M2)scale factor = (A1)  
 *q* = 4 A1 N4



[6]

**18.** (a) Evidence of using Pythagoras (M1)*e.g.* diagram, sin2 *x* + cos2 *x* = 1  
Correct calculation (A1)*e.g.*   
sin *θ* = A1 N3



(b) Evidence of using formula for cos 2*θ* (M1)*e.g.* cos 2*θ* = 2 cos2 *θ* – 1  
Correct substitution/calculation A1*e.g.*   
cos 2*θ* = A1 N2



(c) sin (*θ* + π) = –sin *θ* = A1 N1



[7]

**19.** (a) Attempt to factorise (M1)correct factors (2sin *θ* – 1) (sin *θ* + 1) = 0 A1  
sin *θ* = , sin *θ* = –1 A1A1 N2



(b) other solutions are 150°, 270° A1A1 N1N1

[6]

**20.** (a) When *t* = 1, *l* = 33 + 5 cos 720 (M1)*l* = 33 + 5 = 38 A1 N2

(b) Minimum when cos = –1 (M1)*l*min = 33 – 5 (M1)= 28 A1 N3

(c) 33 = 33 + 5cos720*t* (0 = 5 cos 720*t*) M1720*t* = 90 A1*t* = A1 N1



(d) Evidence of dividing into 360 (M1)  
period = A1 N2



[10]

**21.** ***Note****: Throughout this question, do* ***not*** *accept methods which involve  
finding  .*

(a) Evidence of correct approach A1

*eg* sin ** =



sin ** = AG N0



(b) Evidence of using sin 2** = 2 sin ** cos ** (M1)

= A1



= AG N0



(c) Evidence of using an appropriate formula for cos 2** M1

*eg*



cos 2** = A2 N2



[6]

**22.** (a) For using perimeter = *r* + *r* + arc length (M1)

20 = 2r + *r* A1

AG N0



(b) Finding *A* = (A1)



For setting up equation in *r* M1

Correct simplified equation, or sketch

*eg* 10*r* – *r*2 = 25, *r*2 – 10*r* + 25 = 0 (A1)

*r* = 5 cm A1 N2

[6]

**23.** (a)



Correct asymptotes A1A1 N2

(b) (i) Period = 360 (accept 2) A1 N1

(ii) *f* (90) = 2 A1 N1

(c) 270, 90 A1A1 N1N1

**Notes**: Penalize **1 mark** for any additional values.

Penalize **1 mark** for correct answers given  
 in radians



[6]

**24.** (a) Vertex is (4, 8) A1A1 N2

(b) Substituting 10 = *a*(7  4)2 + 8 M1

*a* = 2 A1 N1

(c) For *y*-intercept, *x* = 0 (A1)

*y* = 24 A1 N2

[6]

**25.** **METHOD 1**

Evidence of correctly substituting into *A* = A1



Evidence of correctly substituting into *l* = *r* A1

For attempting to eliminate one variable … (M1)

leading to a correct equation in one variable A1

*r* = 4 ** = (= 0.524, 30) A1A1 N3



**METHOD 2**

Setting up and equating ratios (M1)

A1A1



Solving gives *r* = 4 A1

*r* = A1



** = A1



*r* = 4 ** = N3



[6]

**26.** *a* = 4, *b* = 2, *c* = A2A2A2 N6



[6]

**27.** **METHOD 1**

Evidence of correctly substituting into *l* = *r* A1

Evidence of correctly substituting into *A* = A1



For attempting to solve these equations (M1)

eliminating one variable correctly A1

*r* = 15 ** = 1.6 (= 91.7) A1A1 N3

**METHOD 2**

Setting up and equating ratios (M1)

A1A1



Solving gives *r* = 15 A1

*r* = 24 A1



** = 1.6 (= 91.7) A1

*r* = 15 ** = 1.6 (= 91.7) N3

[6]

**28.** (a) Evidence of choosing the double angle formula (M1)

*f* (*x*) = 15 sin (6*x*) A1 N2

(b) Evidence of substituting for *f* (*x*) (M1)

*eg* 15 sin 6*x* = 0, sin 3*x* = 0 **and** cos 3*x* = 0

6*x* = 0, , 2

*x* = 0, A1A1A1 N4



[6]

**29.** (a) *p* = 30 A2 2

(b) **METHOD 1**

Period = (M2)  
 = (A1)  
  *q* = 4 A1 4



**METHOD 2**Horizontal stretch of scale factor = (M2)  
scale factor = (A1)  
 *q* = 4 A1 4



[6]

**30.** (a)



(M1)(A1)



(A1)



(A1) (C4)



(b) Arc length (M1)



Arc length = 9 cm (A1) (C2)

**Note:** Penalize a total of (1 mark) for missing units.

[6]

**31.** (a) when (may be implied by a sketch) (A1)



(A1) (C2)



(b) **METHOD 1**

Sketch of appropriate graph(s) (M1)Indicating **correct** points (A1)  
 (A1)(A1) (C2)(C2)



**METHOD 2**



, (A1)(A1)



,



, (A1)(A1) (C2)(C2)



[6]

**32.** Using area of a triangle = *ab* sin *C* (M1)



(A1)(A1)(A1)



**Note:** Accept any letter for Q

sin *Q* = 0.5 (A1)

= 30 or or 0.524 (A1) (C6)



[6]

**33.** (a) *b* = 6 (A1) (C1)

(b)



(A3) (C3)

(c) *x* = 1.05 (accept (1.05, −0.896) ) (correct answer only, no additional  
 solutions) (A2) (C2)

[6]

**34.** (a) 3(1  2 sin2 *x*) + sin *x* = 1 (A1)

6 sin2 *x*  sin *x*  2 = 0 (*p* = 6, *q* = 1, *r* = 2) (A1) (C2)

(b) (3 sin *x*  2)(2 sin *x* + 1) (A1)(A1) (C2)

(c) 4 solutions (A2) (C2)

[6]

**35.** Area of large sector *r*2*θ* = 162 × 1.5 (M1)  
 = 192 (A1)



Area of small sector *r*2*θ* = × 102 × 1.5 (M1)  
 = 75 (A1)



Shaded area = large area – small area = 192 – 75 (M1)  
 = 117 (A1) (C6)

[6]

**36.** (a)  
 (A1)(A1) (C2)



**Note:** Award (A1) for the graph crossing the y-axis between 0.5 and 1, and (A1) for an approximate sine curve crossing the x-axis twice. Do **not** penalize for x >3.14.

(b) (Maximum) *x* = 0.285… (A1)  
 *x* = 0.3 (1 dp) (A1) (C2)



(Minimum) *x* = 1.856… (A1)  
 *x* = 1.9 (1 dp) (A1) (C2)



[6]

**37.** Area of a triangle = × 3 × 4 sin *A* (A1)  
 × 3 × 4 sin *A* = 4.5 (A1)  
sin *A* = 0.75 (A1)  
*A* = 48.6° and *A* = 131° (or 0.848, 2.29 radians) (A1)(A2) (C6)



**Note:** Award (C4) for 48.6° only, (C5) for 131° only.

[6]

**38.** **METHOD 1**

2 cos2 *x* = 2 sin *x* cos *x* (M1)  
2 cos2 *x* – 2 sin *x* cos *x* = 0  
2 cos *x*(cos *x* – sin *x*) = 0 (M1)  
cos *x* = 0, (cos *x* – sin *x*) = 0 (A1)(A1)  
*x* = , *x* = (A1)(A1) (C6)



**METHOD** **2**

Graphical solutions

**EITHER**

for both graphs *y* = 2 cos2 *x*, *y* = sin 2 *x*, (M2)

**OR**

for the graph of *y* = 2 cos2 *x* – sin 2 *x*. (M2)

**THEN**

Points representing the solutions clearly indicated (A1)  
1.57, 0.785 (A1)*x* = , *x* = (A1)(A1) (C6)



**Notes:** If no working shown, award (C4) for one correct answer.  
Award (C2)(C2) for each correct decimal answer 1.57, 0.785.  
Award (C2)(C2) for each correct degree answer 90°, 45°. Penalize a total of [1 mark] for any additional answers.

[6]

**39.** (a) Angle (A1)



(M1)



cm (A1) (C3)



(b) Area (M1)(A1)



7.07 (accept 7.06) (A1) (C3)



**Note:** Penalize once in this question for absence of units.

[6]

**40.** **METHOD 1**

Area sector OAB (M1)



(A1)



ON (A1)



AN (A1)



Area of



(A1)



Shaded area



(A1) (C6)



**METHOD 2**



Area sector (M1)



(A1)



Area (M1)



(A1)



Twice the shaded area (M1)



Shaded area



(A1) (C6)



[6]

**41.** 3 = *p* + *q* cos 0 (M1)  
3 = *p* + *q* (A1)  
–1 = *p* + *q* cos  (M1)  
–1 = *p* – *q* (A1)

(a) *p* = 1 (A1) (C3)

(b) *q* *=* 2 (A1) (C3)

[6]

**42.** **Method 1**



0 (C2)  
1.80 [3 sf] (G2) (C2)  
2.51 [3 sf] (G2) (C2)

**Method 2**

3*x* = ±0.5*x* + 2 (*etc.*) (M1)  
 3.5*x* = 0, 2, 4 **or** 2.5*x* = 0, 2, 4 (A1)  
7*x* = 0, 4, (8) **or** 5*x* = 0, 4, (8) (A1)  
 *x* *=* 0, **or** *x* *=* 0, (A1)(A1)(A1)  
 *x* *=* 0, , (C2)(C2)(C2)



[6]

**43.** (a) area of sector ΑΒDC = π(2)2 = π (A1)  
area of segment BDCP = π – area of ABC (M1)  
 = π – 2 (A1) (C3)



(b) BP = (A1)  
area of semicircle of radius BP = π()2 = π (A1)  
area of shaded region = π – (π – 2) = 2 (A1) (C3)



[6]

**44.** (a) (i) A is (A1)(A1) (C2)



(ii) B is (0, –4) (A1)(A1) (C2)

**Note:** In each of parts (i) and (ii), award C1 if A and B are interchanged, C1 if intercepts given instead of coordinates.

(b) Area = × 4 × (M1)  
 = (= 2.67) (A1) (C2)



[6]

**45.** (a) (3 sin *x* – 2)(sin *x* – 3) (A1)(A1) (C2)

**Note**: Award A1 if 3x2 – 11x + 6 correctly factorized to give  
(3x – 2)(x – 3) (or equivalent with another letter).

(b) (i) (3 sin *x* – 2)(sin *x* – 3) = 0  
sin *x* = sin *x* = 3 (A1)(A1) (C2)



(ii) *x* = 41.8°, 138° (A1)(A1) (C2)

**Notes**: Penalize [1 mark] for any extra answers and [1 mark] for answers in radians.  
ie Award A1 A0 for 41.8°, 138° and any extra answers.  
Award A1 A0 for 0.730, 2.41.  
Award A0 A0 for 0.730, 2.41 and any extra answers.

[6]

**46.** (a) *l* = *r* or ACB = 2 × OA (M1)  
 = 30 cm (A1) (C2)

(b) (obtuse) = 2 – 2 (A1)  
Area = * r 2* = (2 – 2)(15)2 (M1)(A1)  
 = 482 cm2 (3 sf) (A1) (C4)



[6]

**47.**



(M1)(A2)  
**OR**2.5 × 20 = 50 (M1)(A1)  
2.5 × 32 = 80 (A1)

*d*2 = 502 + 802 – 2 × 50 × 80 × cos 70° (M1)(A1)  
*d* = 78.5 km (A1) (C6)

[6]

**48.** (a) (i) –1 (A1) (C1)

(ii) 4 (accept 720°) (A2) (C2)

(b)



(G1)  
number of solutions: 4 (A2) (C3)

[6]

**49.**

|  |  |  |  |
| --- | --- | --- | --- |
| Statement | (a) Is the statement true for all real numbers *x*? (Yes*/*No) | (b) If not true, example |  |
| A | No | *x* = –l (log10 0.1 = –1) | (a) (A3) (C3) |
| B | No | *x* = 0 (cos 0 = 1) | (b) (A3) (C3) |
| C | Yes | N*/*A |  |

**Notes**: (a) Award (A1) for each correct answer.

(b) Award (A) marks for statements A and B only if NO in column (a).  
Award (A2) for a correct counter example to statement A, (A1) for a correct counter example to statement B (ignore other incorrect examples).  
**Special Case for statement C:**

Award (A1) if candidates write NO, and give a valid reason (eg arctan 1 = ).



[6]

**50.** Using sine rule: (M1)(A1)  
 sin *B =* sin 48° = 0.5308… (M1)  
 *B =* arcsin (0.5308) = 32.06° (M1)(A1)  
 = 32° (nearest degree) (A1) (C6)



**Note:** Award a maximum of [5 marks] if candidates give the answer in radians (0.560).

[6]

**51.** (a) *x* is an acute angle => cos *x* is positive. (M1)  
cos2 *x* + sin2 *x* = 1 => cos *x* = (M1)  
=> cos *x* = (A1)  
 = (= ) (A1) (C4)



(b) cos 2*x* = 1 – 2 sin2 *x* = 1 – 2 (M1)  
= (A1) (C2)



**Notes:** (a) Award (M1)(M0)(A1)(A0) for  
cos = 0.943.  
(b) Award (M1)(A0) for cos = 0.778.



[6]

**52.** (a) 2 sin2 *x* = 2(1 – cos2 *x*) = 2 – 2 cos2 *x* = l + cos *x* (M1)  
=> 2 cos2 *x* + cos *x* – l = 0 (A1) (C2)

**Note:** Award the first (M1) for replacing sin2 x by   
1 – cos2 x.

(b) 2 cos2 *x* + cos *x* – 1 = (2 cos *x* – 1)(cos *x* +1) (A1) (C1)

(c) cos *x* = or cos *x* = –l   
=> *x* = 60°, 180° or 300° (A1)(A1)(A1) (C3)



**Note:** Award (A1)(A1)(A0) if the correct answers are given in radians (ie , ,, or 1.05, 3.14, 5.24)



[6]

**53.** (a) The smallest angle is opposite the smallest side.  
cos *θ* = (M1)  
 = = 0.7857  
Therefore, *θ* = 38.2° (A1)(C2)



(b) Area = × 8 × 7 × sin 38.2° (M1)  
 = 17.3 cm2 (A1)(C2)



[4]

**54.** (a) 3 sin2 *x* + 4 cos *x* = 3(1 – cos2 *x*) + 4cos *x*  
 = 3 – 3 cos2 + 4 cos *x* (A1)(C1)

(b) 3 sin2 *x* + 4 cos *x* – 4 = 0 3 – 3 cos2 *x* + 4 cos *x* – 4 = 0  
  3 cos2 *x* – 4 cos *x* + 1 = 0 (A1)  
 (3 cos *x* – 1)(cos *x* – 1) = 0  
 cos *x* = or cos *x* = 1  
 *x* = 70.5° or *x* = 0° (A1)(A1)(C3)



**Note:** Award (C1) for each correct radian answer, ie x = 1.23 or x = 0.

[4]

**55.** = 90° (A1)AT =   
=



= 60° = (A1)  
Area = area of triangle – area of sector  
 = × 6 × – × 6 × 6 × (M1)  
 = 12.3 cm2 (or – 6) (A1) (C4)



**OR**

= 60° (A1)  
Area of  = × 6 × 12 × sin 60 (A1)  
Area of sector = × 6 × 6 × (A1)  
Shaded area = – 6 = 12.3 cm2 (3 sf) (A1) (C4)



[4]

**56.** (a) Area = (152)(2) (M1)  
= 225 (cm2) (A1) (C2)



(b) Area ∆OAB = 152 sin 2 = 102.3 (A1)  
Area = 225 – 102.3 = 122.7 (cm2)  
 = 123 (3 sf) (A1) (C2)



[4]

**57.** (a) (M1)  
 = 0.901  
 > 90°  = 180° – 64.3° = 115.7°  
 = 116 (3 sf) (A1) (C2)



(b) In Triangle 1, = 64.3°  
 = 180° – (64.3° + 50°)  
= 65.7° (A1)  
Area = (20)(17) sin 65.7° = 155 (cm2) (3 sf) (A1) (C2)



[4]

**58.** **METHOD** **1**

The value of cosine varies between –1 and +1. Therefore:  
*t* = 0  *a* + *b* = 14.3  
*t* = 6  *a* – *b* = 10.3  
  2*a* = 24.6  *a* = 12.3 (A1) (C1)  2*b* = 4.0  *b* = 2 (A1) (C1)Period = 12 hours  = 2π (M1) *k* = 12 (A1) (C2)



**METHOD** **2**



From consideration of graph: Midpoint = *a* = 12.3 (A1) (C1)  
 Amplitude = *b* = 2 (A1) (C1)  
 Period = = 12 (M1)  
  *k* = 12 (A1) (C2)



[4]

**59.**



cos (M1)(A1)  
 = arccos(0.0625) (A1)  
 86° (A1)



[4]

**60.** (a) 2 cos2 *x* + sin *x* = 2(1 – sin2 *x*) + sin *x* = 2 – 2 sin2 *x* + sin *x* (A1)

(b) 2 cos2 *x* + sin *x* = 2  
 2 – 2 sin2 *x* + sin *x* = 2  
sin *x* – 2 sin2 *x* = 0  
sin *x*(1 – 2 sin *x*) = 0  
sin *x* = 0 or sin *x* = (M1)  
sin *x* = 0  *x* = 0 or  (0° or 180°) (A1)



**Note:** Award (A1) for both answers.

sin *x* =  *x* = or (30° or 150°) (A1)



**Note:** Award (A1) for both answers.

[4]

**61.** 3 cos *x* = 5 sin *x*  
 (M1)  
 tan *x* = 0.6 (A1)  
*x* = 31° or *x* = 211° (to the nearest degree) (A1)(A1) (C2)(C2)



**Note:** Deduct [1 mark] if there are more than two answers.

[4]

**62.** sin *A* =  cos *A* =  (A1)  
But *A* is obtuse  cos *A* = – (A1)  
sin 2*A* = 2 sin *A* cos *A* (M1)  
= 2 ×   
= – (A1) (C4)



[4]

**63.** (a)  
  
Acute angle 30° (M1)



**Note:** Award the (M1) for 30° and/or quadrant diagram/graph seen.

2nd quadrant since sine positive and cosine negative  
 ** = 150° (A1) (C2)

(b) tan 150° = –tan 30° **or** tan 150° = (M1)  
tan 150° = – (A1) (C2)



[4]

**64.** (a) = tan 36°  
 PQ  29.1 m (3 sf) (A1) (C1)



(b)  
  
 = 80° (A1)  
 (M1)



**Note:** Award (M1) for correctly substituting.

 AB = 41 9. m (3 sf) (A1) (C3)

[4]

**65.** Perimeter = 5(2π – 1) + 10 (M1)(A1)(A1)

**Note:** Award (M1) for working in radians; (A1) for 2π – 1; (A1) for +10.

= (10π + 5) cm (= 36.4, to 3 sf) (A1) (C4)

[4]

**66.** From sketch of graph *y* = 4 sin (M2)  
or by observing sin **  1.  
*k* > 4, *k* < –4 (A1)(A1) (C2)(C2)



[4]

**67.** ***Note:*** *Award (M1) for identifying the largest angle*.

cos  = (M1)  
= – (A1)  
  = 101.5° (A1)



**OR** Find other angles first

 = 44.4°  = 34.0° (M1)

  = 101.6° (A1)(A1) (C4)

**Note:** Award (C3) if not given to the correct accuracy.

[4]

**68.** *AB* = *r*  
= (M1)(A1)  
= 21.6 × (A1)  
= 8 cm (A1)



**OR** × (5.4)2** = 21.6  
 ** = (= 1.481 radians) (M1)  
*AB* = *r* (A1)  
= 5.4 × (M1)  
= 8 cm (A1) (C4)



[4]

**69.** tan2 *x* = (M1)  
 tan *x* =  (M1)  
 *x* = 30° or *x* = 150° (A1)(A1) (C2)(C2)



[4]

**70.** *h* = *r* so 2*r*2 = 100  *r*2 = 50 (M1)  
*l* = 10** = 2*r* (M1)  
 ** = (A1)  
=   
** =  = 4.44 (3sf) (A1) (C4)



**Note:** Accept either answer.

[4]