

C3 FUNCTIONS

Answers - Worksheet B

- 1** **a** $= g(5) = -3$ **b** $= g(6) = -4$ **c** $= f(5) = 17$ **d** $= h(9) = 86$
e $= g(-3) = 5$ **f** $= f(-1) = -7$ **g** $= h(-6) = 41$ **h** $= f\left(\frac{29}{4}\right) = 26$
- 2** **a** $= f(\ln 20) = 17.0$ **b** $= g(\ln 3) = 0.455$ **c** $= f(\cos 5) = 3.42$ **d** $= g(\cos(-4)) = 0.794$
e $= g\left(\frac{43}{4}\right) = -0.243$ **f** $= h(\cos 6.7) = -0.0895$ **g** $= h(\ln 50) = 1.36$ **h** $= h(0.5) = -0.693$
- 3** **a** $= f(1 - 3x)$
 $= 2(1 - 3x) + 1$
 $fg : x \rightarrow 3 - 6x,$
 $x \in \mathbb{R}$ **b** $= f(2x + 1)$
 $= 2(2x + 1) + 1$
 $ff : x \rightarrow 4x + 3,$
 $x \in \mathbb{R}$ **c** $= f(x^2 + 4)$
 $= 2(x^2 + 4) + 1$
 $fh : x \rightarrow 2x^2 + 9,$
 $x \in \mathbb{R}$ **d** $= h(2x + 1)$
 $= (2x + 1)^2 + 4$
 $hf : x \rightarrow 4x^2 + 4x + 5,$
 $x \in \mathbb{R}$
- e** $= g(x^2 + 4)$
 $= 1 - 3(x^2 + 4)$
 $gh : x \rightarrow -3x^2 - 11,$
 $x \in \mathbb{R}$ **f** $= g(1 - 3x)$
 $= 1 - 3(1 - 3x)$
 $gg : x \rightarrow 9x - 2,$
 $x \in \mathbb{R}$ **g** $= h(1 - 3x)$
 $= (1 - 3x)^2 + 4$
 $hg : x \rightarrow 9x^2 - 6x + 5,$
 $x \in \mathbb{R}$ **h** $= g(2x + 1)$
 $= 1 - 3(2x + 1)$
 $gf : x \rightarrow -6x - 2,$
 $x \in \mathbb{R}$
- 4** **a** $= g(4 - x)$
 $= e^{4-x}$
 $gf : x \rightarrow e^{4-x},$
 $x \in \mathbb{R}$ **b** $= h(e^x)$
 $= 2(e^x)^2 + 7$
 $hg : x \rightarrow 2e^{2x} + 7,$
 $x \in \mathbb{R}$ **c** $= f(2x^2 + 7)$
 $= 4 - (2x^2 + 7)$
 $fh : x \rightarrow -2x^2 - 3,$
 $x \in \mathbb{R}$ **d** $= g(e^x)$
 $= e^{e^x}$
 $gg : x \rightarrow e^{e^x},$
 $x \in \mathbb{R}$
- e** $= g(2x^2 + 7)$
 $= e^{2x^2+7}$
 $gh : x \rightarrow e^{2x^2+7},$
 $x \in \mathbb{R}$ **f** $= f(4 - x)$
 $= 4 - (4 - x)$
 $ff : x \rightarrow x,$
 $x \in \mathbb{R}$ **g** $= f(e^x)$
 $= 4 - e^x$
 $fg : x \rightarrow 4 - e^x,$
 $x \in \mathbb{R}$ **h** $= h(4 - x)$
 $= 2(4 - x)^2 + 7$
 $hf : x \rightarrow 2x^2 - 16x + 39,$
 $x \in \mathbb{R}$
- 5** **a** $ff(x) = f(5x - 3)$
 $= 5(5x - 3) - 3$
 $= 25x - 18$
 $25x - 18 = -8$
 $x = \frac{2}{5}$ **b** $hf(x) = h(5x - 3)$
 $= \frac{1}{5x-3-2}$
 $= \frac{1}{5x-5}$
 $\frac{1}{5x-5} = 2$
 $1 = 10x - 10$
 $x = \frac{11}{10}$ **c** $gf(x) = g(5x - 3)$
 $= 3(5x - 3)^2 + 1$
 $= 75x^2 - 90x + 28$
 $75x^2 - 90x + 28 = 28$
 $15x(5x - 6) = 0$
 $x = 0, \frac{6}{5}$ **d** $hg(x) = h(3x^2 + 1)$
 $= \frac{1}{3x^2+1-2}$
 $= \frac{1}{3x^2-1}$
 $\frac{1}{3x^2-1} = \frac{1}{2}$
 $3x^2 - 1 = 2$
 $x^2 = 1$
 $x = \pm 1$
- e** $fh(x) = f\left(\frac{1}{x-2}\right)$
 $= \frac{5}{x-2} - 3$
 $\frac{5}{x-2} - 3 = 7$
 $\frac{5}{x-2} = 10$
 $5 = 10x - 20$
 $x = \frac{5}{2}$ **f** $fg(x) = f(3x^2 + 1)$
 $= 5(3x^2 + 1) - 3$
 $= 15x^2 + 2$
 $15x^2 + 2 = 32$
 $x^2 = 2$
 $x = \pm\sqrt{2}$ **g** $gh(x) = g\left(\frac{1}{x-2}\right)$
 $= \frac{3}{(x-2)^2} + 1$
 $\frac{3}{(x-2)^2} + 1 = 4$
 $(x-2)^2 = 1$
 $x-2 = \pm 1$
 $x = 1, 3$ **h** $hh(x) = h\left(\frac{1}{x-2}\right)$
 $= \frac{1}{\frac{1}{x-2}-2} = \frac{x-2}{1-2(x-2)}$
 $= \frac{x-2}{5-2x}$
 $\frac{x-2}{5-2x} = -2$
 $x-2 = -10 + 4x$
 $x = \frac{8}{3}$

6

a	$gh(x) = g(e^x)$ $= 3 + 2e^x$ $3 + 2e^x = 9$ $e^x = 3$ $x = \ln 3$ $x = 1.10$	b	$fg(x) = f(3 + 2x)$ $= \ln(3 + 2x)$ $\ln(3 + 2x) = 3.6$ $3 + 2x = e^{3.6}$ $x = \frac{1}{2}(e^{3.6} - 3)$ $x = 16.80$	c	$hg(x) = h(3 + 2x)$ $= e^{3+2x}$ $e^{3+2x} = 4$ $3 + 2x = \ln 4$ $x = \frac{1}{2}(\ln 4 - 3)$ $x = -0.81$	d	$gf(x) = g(\ln x)$ $= 3 + 2 \ln x$ $3 + 2 \ln x = 10.4$ $\ln x = 3.7$ $x = e^{3.7}$ $x = 40.45$
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7

a $g(x) > 0$

b $fg(x) = f(e^x) = \frac{e^x + 1}{5}$

$$\frac{e^x + 1}{5} = 17$$

$$e^x + 1 = 85$$

$$e^x = 84$$

$$x = \ln 84 = 4.43 \text{ (3sf)}$$

8

a $f(4) = 7$

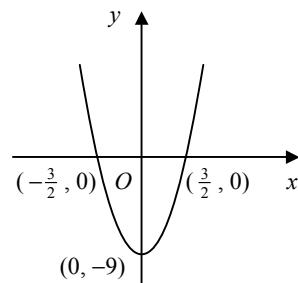
b $gf(x) = g(4x - 9) = (4x - 9)^2$

$$\therefore (4x - 9)^2 = 25$$

$$4x - 9 = \pm 5$$

$$x = \frac{1}{4}(9 \pm 5) = 1, \frac{7}{2}$$

c $fg(x) = f(x^2) = 4x^2 - 9$



9

a	$= g(1) = 4$	b	$= h(2) = e^3$	c	$= g(e^{-3}) = 1$	d	$= f(\tan 1) = 74.7 \text{ (3sf)}$
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e

$$= h(\tan 0.2) = 0.552 \text{ (3sf)}$$

f

$$= f(4 + \ln 7) = -0.351 \text{ (3sf)}$$

g

$$= h(e^{-\frac{1}{2}}) = 1.24 \text{ (3sf)}$$

h

$$= f(4 + e) = 0.465 \text{ (3sf)}$$

10

a	$= f(4x + 1) = 3e^{4x+1} + 2$	b	$= g(3e^x + 2) = 4(3e^x + 2) + 1$	c	$= h(3e^x + 2) = \frac{1}{3e^x + 2 + 1}$	d	$= g(4x + 1) = 4(4x + 1) + 1$
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$fg : x \rightarrow 3e^{4x+1} + 2, \quad gf : x \rightarrow 12e^x + 9,$

$x \in \mathbb{R} \quad x \in \mathbb{R} \quad x \in \mathbb{R} \quad x \in \mathbb{R}$

$hf : x \rightarrow \frac{1}{3e^x + 3}, \quad gg : x \rightarrow 16x + 5,$

e	$= h(4x + 1) = \frac{1}{4x+1+1}$	f	$= g\left(\frac{1}{x+1}\right) = \frac{4}{x+1} + 1$	g	$= h\left(\frac{1}{x+1}\right) = \frac{1}{\frac{1}{x+1}+1} = \frac{x+1}{1+x+1}$	h	$= g(16x + 5) = 4(16x + 5) + 1$
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$hg : x \rightarrow \frac{1}{4x+2}, \quad gh : x \rightarrow \frac{x+5}{x+1}, \quad hh : x \rightarrow \frac{x+1}{x+2}, \quad ggg : x \rightarrow 64x + 21,$

$x \in \mathbb{R}, x \neq -\frac{1}{2} \quad x \in \mathbb{R}, x \neq -1 \quad x \in \mathbb{R}, x \neq -1, -2 \quad x \in \mathbb{R}$

11 **a** $fh(x) = f\left(\frac{x+1}{3}\right)$

$$\begin{aligned} &= \sqrt{\frac{x+1}{3} + 4} \\ &= \sqrt{\frac{x+13}{3}} \\ &\sqrt{\frac{x+13}{3}} = 3 \\ &\frac{x+13}{3} = 9 \\ &x+13 = 27 \\ &x = 14 \\ &x+13 = 27 \end{aligned}$$

b $fg(x) = f(e^{1+2x})$

$$\begin{aligned} &= \sqrt{e^{1+2x} + 4} \\ &\sqrt{e^{1+2x} + 4} = 7 \\ &e^{1+2x} + 4 = 49 \\ &e^{1+2x} = 45 \\ &1+2x = \ln 45 \\ &x = \frac{1}{2}(\ln 45 - 1) \\ &x = 1.40 \text{ (3sf)} \end{aligned}$$

c $gh(x) = g\left(\frac{x+1}{3}\right)$

$$\begin{aligned} &= e^{1+\frac{2(x+1)}{3}} \\ &= e^{\frac{2x+5}{3}} \\ &e^{\frac{2x+5}{3}} = 11 \\ &\frac{2x+5}{3} = \ln 11 \\ &2x+5 = 3 \ln 11 \\ &x = \frac{1}{2}(3 \ln 11 - 5) \\ &x = 1.10 \text{ (3sf)} \end{aligned}$$

d $hh(x) = h\left(\frac{x+1}{3}\right)$

$$\begin{aligned} &= \frac{\frac{x+1}{3} + 1}{3} \\ &= \frac{x+4}{9} \\ &\frac{x+4}{9} = \frac{2}{3} \\ &x+4 = 6 \\ &x = 2 \\ &3x+12 = 18 \end{aligned}$$

e $hg(x) = h(e^{1+2x})$

$$\begin{aligned} &= e^{1+2x+1} \\ &= e^{1+2x+1} \\ &e^{1+2x+1} = 1.2 \\ &e^{1+2x} = 2.6 \\ &1+2x = \ln 2.6 \\ &x = \frac{1}{2}(\ln 2.6 - 1) \\ &x = -0.0222 \text{ (3sf)} \end{aligned}$$

f $hf(x) = h(\sqrt{x+4})$

$$\begin{aligned} &= \frac{\sqrt{x+4}+1}{3} \\ &\frac{\sqrt{x+4}+1}{3} = \frac{1}{2} \\ &\sqrt{x+4}+1 = \frac{3}{2} \\ &\sqrt{x+4} = \frac{1}{2} \\ &x+4 = \frac{1}{4} \\ &x = -3\frac{3}{4} \end{aligned}$$

g $ff(x) = f(\sqrt{x+4})$

$$\begin{aligned} &= \sqrt{\sqrt{x+4}+4} \\ &\sqrt{\sqrt{x+4}+4} = 3 \\ &\sqrt{\sqrt{x+4}+4} = 5 \\ &x+4 = 25 \\ &x = 21 \end{aligned}$$

h $ggh(x) = g\left(\frac{x+4}{9}\right)$

$$\begin{aligned} &= e^{1+\frac{2(x+4)}{9}} \\ &= e^{\frac{2x+17}{9}} \\ &e^{\frac{2x+17}{9}} = \frac{1}{2} \\ &\frac{2x+17}{9} = \ln \frac{1}{2} \\ &x = \frac{1}{2}(9 \ln \frac{1}{2} - 17) \\ &x = -11.6 \text{ (3sf)} \end{aligned}$$

12 **a** $h(x) = fg(x)$

b $h(x) = gf(x)$

c $h(x) = gg(x)$

d $h(x) = ff(x)$

e $h(x) = gff(x)$

f $h(x) = gfg(x)$

13 **a** $j(x) = fg(x)$

b $j(x) = hf(x)$

c $j(x) = gh(x)$

d $j(x) = gg(x)$

e $j(x) = fhg(x)$

f $j(x) = hfg(x)$

14 **a** $gf(x) = g(5^x - 7)$

$$\begin{aligned} &= 2(5^x - 7) + 3 \\ &= 2(5^x) - 11 \\ &gf : x \rightarrow 2(5^x) - 11, x \in \mathbb{R} \end{aligned}$$

b $2(5^x) - 11 = 10$

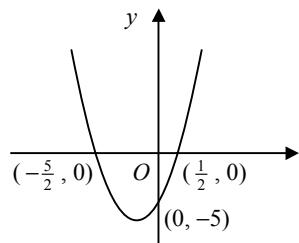
$$\begin{aligned} 5^x &= \frac{21}{2} \\ x &= \frac{\ln \frac{21}{2}}{\ln 5} = 1.46 \text{ (3sf)} \end{aligned}$$

15 **a** $gf(x) = g[2(x+1)] = [2(x+1)]^2 - 9$

$gf : x \rightarrow 4x^2 + 8x - 5, x \in \mathbb{R}$

range: $gf(x) \geq -9$

b $gf(x) = (2x+5)(2x-1)$



c $gf(x) - 2f(x) = a$

$$\begin{aligned} 4x^2 + 8x - 5 - 2[2(x+1)] &= a \\ 4x^2 + 4x - (a+9) &= 0 \\ \text{no real roots } \therefore b^2 - 4ac < 0 & \end{aligned}$$

$$\begin{aligned} 16 + 16(a+9) &< 0 \\ 1 + a + 9 &< 0 \\ a &< -10 \end{aligned}$$