

6	a	$gh(x) = g(e^x)$	b	$fg(x) = f(3 + 2x)$	c	$hg(x) = h(3 + 2x)$	d	$gf(x) = g(\ln x)$
		$= 3 + 2e^x$		$= \ln(3 + 2x)$		$= e^{3+2x}$		$= 3 + 2 \ln x$
		$3 + 2e^x = 9$		$\ln(3 + 2x) = 3.6$		$e^{3+2x} = 4$		$3 + 2 \ln x = 10.4$
		$e^x = 3$		$3 + 2x = e^{3.6}$		$3 + 2x = \ln 4$		$\ln x = 3.7$
		$x = \ln 3$		$x = \frac{1}{2}(e^{3.6} - 3)$		$x = \frac{1}{2}(\ln 4 - 3)$		$x = e^{3.7}$
	$x = 1.10$		$x = 16.80$		$x = -0.81$		$x = 40.45$	

7 a $g(x) > 0$

b $fg(x) = f(e^x) = \frac{e^x + 1}{5}$

$$\frac{e^x + 1}{5} = 17$$

$$e^x + 1 = 85$$

$$e^x = 84$$

$$x = \ln 84 = 4.43 \text{ (3sf)}$$

8 a $= f(4) = 7$

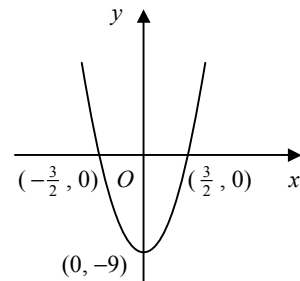
b $gf(x) = g(4x - 9) = (4x - 9)^2$

$$\therefore (4x - 9)^2 = 25$$

$$4x - 9 = \pm 5$$

$$x = \frac{1}{4}(9 \pm 5) = 1, \frac{7}{2}$$

c $fg(x) = f(x^2) = 4x^2 - 9$



9 a $= g(1) = 4$

b $= h(2) = e^3$

c $= g(e^{-3}) = 1$

d $= f(\tan 1) = 74.7 \text{ (3sf)}$

e $= h(\tan 0.2)$
 $= 0.552 \text{ (3sf)}$

f $= f(4 + \ln 7)$
 $= -0.351 \text{ (3sf)}$

g $= h(e^{-\frac{1}{2}})$
 $= 1.24 \text{ (3sf)}$

h $= f(4 + e)$
 $= 0.465 \text{ (3sf)}$

10 a $= f(4x + 1)$
 $= 3e^{4x+1} + 2$

b $= g(3e^x + 2)$
 $= 4(3e^x + 2) + 1$

c $= h(3e^x + 2)$
 $= \frac{1}{3e^x + 2 + 1}$

d $= g(4x + 1)$
 $= 4(4x + 1) + 1$

$fg : x \rightarrow 3e^{4x+1} + 2,$
 $x \in \mathbb{R}$

$gf : x \rightarrow 12e^x + 9,$
 $x \in \mathbb{R}$

$hf : x \rightarrow \frac{1}{3e^x + 3},$
 $x \in \mathbb{R}$

$gg : x \rightarrow 16x + 5,$
 $x \in \mathbb{R}$

e $= h(4x + 1)$
 $= \frac{1}{4x + 1 + 1}$

f $= g\left(\frac{1}{x+1}\right)$
 $= \frac{4}{x+1} + 1$

g $= h\left(\frac{1}{x+1}\right)$
 $= \frac{1}{\frac{1}{x+1} + 1}$

h $= g(16x + 5)$
 $= 4(16x + 5) + 1$

$hg : x \rightarrow \frac{1}{4x + 2},$
 $x \in \mathbb{R}, x \neq -\frac{1}{2}$

$\frac{4+x+1}{x+1}$
 $gh : x \rightarrow \frac{x+5}{x+1},$
 $x \in \mathbb{R}, x \neq -1$

$\frac{x+1}{1+x+1}$
 $hh : x \rightarrow \frac{x+1}{x+2},$
 $x \in \mathbb{R}, x \neq -1, -2$

$ggg : x \rightarrow 64x + 21,$
 $x \in \mathbb{R}$

11 a $fh(x) = f\left(\frac{x+1}{3}\right)$
 $= \sqrt{\frac{x+1}{3} + 4}$
 $= \sqrt{\frac{x+13}{3}}$
 $\sqrt{\frac{x+13}{3}} = 3$
 $\frac{x+13}{3} = 9$
 $x + 13 = 27$
 $x = 14$

b $fg(x) = f(e^{1+2x})$
 $= \sqrt{e^{1+2x} + 4}$
 $\sqrt{e^{1+2x} + 4} = 7$
 $e^{1+2x} + 4 = 49$
 $e^{1+2x} = 45$
 $1 + 2x = \ln 45$
 $x = \frac{1}{2}(\ln 45 - 1)$
 $x = 1.40$ (3sf)

c $gh(x) = g\left(\frac{x+1}{3}\right)$
 $= e^{1 + \frac{2(x+1)}{3}}$
 $= e^{\frac{2x+5}{3}}$
 $e^{\frac{2x+5}{3}} = 11$
 $\frac{2x+5}{3} = \ln 11$
 $2x + 5 = 3 \ln 11$
 $x = \frac{1}{2}(3 \ln 11 - 5)$
 $x = 1.10$ (3sf)

d $hh(x) = h\left(\frac{x+1}{3}\right)$
 $= \frac{\frac{x+1}{3} + 1}{3}$
 $= \frac{x+1+3}{9}$
 $= \frac{x+4}{9}$
 $\frac{x+4}{9} = \frac{2}{3}$
 $3x + 12 = 18$
 $x = 2$

e $hg(x) = h(e^{1+2x})$
 $= \frac{e^{1+2x} + 1}{3}$
 $\frac{e^{1+2x} + 1}{3} = 1.2$
 $e^{1+2x} = 2.6$
 $1 + 2x = \ln 2.6$
 $x = \frac{1}{2}(\ln 2.6 - 1)$
 $x = -0.0222$ (3sf)

f $hf(x) = h(\sqrt{x+4})$
 $= \frac{\sqrt{x+4} + 1}{3}$
 $\frac{\sqrt{x+4} + 1}{3} = \frac{1}{2}$
 $\sqrt{x+4} = \frac{1}{2}$
 $x + 4 = \frac{1}{4}$
 $x = -3\frac{3}{4}$

g $ff(x) = f(\sqrt{x+4})$
 $= \sqrt{\sqrt{x+4} + 4}$
 $\sqrt{\sqrt{x+4} + 4} = 3$
 $\sqrt{x+4} = 5$
 $x + 4 = 25$
 $x = 21$

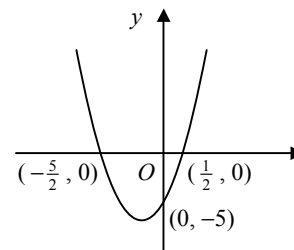
h $ghh(x) = g\left(\frac{x+4}{9}\right)$
 $= e^{1 + \frac{2(x+4)}{9}}$
 $= e^{\frac{2x+17}{9}}$
 $e^{\frac{2x+17}{9}} = \frac{1}{2}$
 $\frac{2x+17}{9} = \ln \frac{1}{2}$
 $x = \frac{1}{2}(9 \ln \frac{1}{2} - 17)$
 $x = -11.6$ (3sf)

12 a $h(x) = fg(x)$ **b** $h(x) = gf(x)$ **c** $h(x) = gg(x)$
d $h(x) = ff(x)$ **e** $h(x) = gff(x)$ **f** $h(x) = gfg(x)$

13 a $j(x) = fg(x)$ **b** $j(x) = hf(x)$ **c** $j(x) = gh(x)$
d $j(x) = gg(x)$ **e** $j(x) = fhg(x)$ **f** $j(x) = hfg(x)$

14 a $gf(x) = g(5^x - 7)$
 $= 2(5^x - 7) + 3$
 $= 2(5^x) - 11$
 $gf : x \rightarrow 2(5^x) - 11, x \in \mathbb{R}$
b $2(5^x) - 11 = 10$
 $5^x = \frac{21}{2}$
 $x = \frac{\ln \frac{21}{2}}{\ln 5} = 1.46$ (3sf)

15 a $gf(x) = g[2(x+1)] = [2(x+1)]^2 - 9$
 $gf : x \rightarrow 4x^2 + 8x - 5, x \in \mathbb{R}$
 range: $gf(x) \geq -9$
b $gf(x) = (2x+5)(2x-1)$



c $gf(x) - 2f(x) = a$
 $4x^2 + 8x - 5 - 2[2(x+1)] = a$
 $4x^2 + 4x - (a+9) = 0$
 no real roots $\therefore b^2 - 4ac < 0$
 $16 + 16(a+9) < 0$
 $1 + a + 9 < 0$
 $a < -10$