

## Calculations with Logarithms – Essential Review

► exponents and logarithms ◀

$$a^x = b \Leftrightarrow x = \log_a b$$

(answers included)

### Exercises

1. (a) What is the value of  $x$  for  $2^x$  to be equal to:
- (i) 4            (ii) 32            (iii)  $\frac{1}{2}$             (iv) 1            (v)  $\frac{1}{16}$
- (b) What is the value of  $x$  for  $3^x$  to be equal to:
- (i) 9            (ii)  $\frac{1}{9}$             (iii)  $\frac{1}{81}$             (iv) 1            (v) 0
- (c) What is the value of  $x$  for  $\left(\frac{1}{2}\right)^x$  to be equal to:
- (i) 2            (ii) 4            (iii)  $\frac{1}{2}$             (iv) 1            (v)  $\frac{1}{128}$
- (d) What is the value of  $x$  for  $(\sqrt{2})^x$  to be equal to:
- (i) 2            (ii) 4            (iii)  $2\sqrt{2}$             (iv)  $\frac{1}{2\sqrt{2}}$             (v) 1
2. (a) Write each of the following in the form  $10^k$
- (i) 10            (ii) 100            (iii)  $\frac{1}{10}$             (iv) 1            (v)  $100\sqrt{10}$
- (b) Write each of the following in the form  $2^k$
- (i) 16            (ii)  $\frac{1}{16}$             (iii) 1            (iv)  $\frac{1}{\sqrt{2}}$             (v) 0
- (c) Write each in the form  $3^k$
- (i) 3            (ii) 9            (iii)  $\frac{1}{9}$             (iv)  $\frac{1}{\sqrt{27}}$             (v)  $-9$
- (d) Write each in the form  $(\sqrt{2})^k$
- (i) 2            (ii) 4            (iii)  $4\sqrt{2}$             (iv)  $\frac{1}{2}$             (v)  $\frac{1}{8\sqrt{2}}$

## Calculations with Logarithms – Essential Review

3. (a) What must  $a$  be for  $\log a$  to be equal to:

- (i) 1            (ii) 2            (iii)  $-1$             (iv) 0            (v)  $\frac{1}{2}$

(b) What must  $a$  be for  $\log_2 a$  to be equal to:

- (i) 2            (ii) 3            (iii)  $-4$             (iv)  $-\frac{1}{2}$             (v) 0

(c) What must  $a$  be for  $\log_{\frac{1}{2}} a$  to be equal to:

- (i) 2            (ii) 1            (iii) 0            (iv)  $-1$             (v) 5

(d) What must  $a$  be for  $\log_{\sqrt{2}} a$  to be equal to:

- (i) 1            (ii)  $-2$             (iii) 3            (iv) 0            (v)  $\frac{1}{2}$

4. (a) State the value of

- (i)  $\log_2 2$             (ii)  $\log_2 4$             (iii)  $\log_2 2^{17}$             (iv)  $\log_2 \sqrt{2}$             (v)  $\log_2 1$

(b) State the value of

- (i)  $\log_3 27$             (ii)  $\log_3 81$             (iii)  $\log_3 \sqrt{3}$             (iv)  $\log_3 \sqrt{27}$             (v)  $\log_2 0$

(c) State the value of

- (i)  $\log_{\frac{1}{3}} 3$             (ii)  $\log_{\frac{1}{3}} 9$             (iii)  $\log_{\frac{1}{3}} \frac{1}{3}$             (iv)  $\log_{\frac{1}{3}} 1$             (v)  $\log_{\frac{1}{3}} \frac{1}{\sqrt{3}}$

(d) State the value of

- (i)  $\log_{\frac{1}{\sqrt{3}}} \frac{1}{\sqrt{3}}$             (ii)  $\log_{\frac{1}{\sqrt{3}}} \frac{1}{3}$             (iii)  $\log_{\frac{1}{\sqrt{3}}} \frac{1}{3\sqrt{3}}$             (iv)  $\log_{\frac{1}{\sqrt{3}}} 3\sqrt{3}$             (v)  $\log_{\frac{1}{\sqrt{3}}} 0$

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5. (a) State the value of

(i)  $\log_8 64$       (ii)  $\log_4 64$       (iii)  $\log_2 64$       (iv)  $\log_{\sqrt{2}} 64$       (v)  $\log_2 8^k$

(b) State the value of

(i)  $\log_{81} 81$       (ii)  $\log_9 81$       (iii)  $\log_3 81$       (iv)  $\log_{\frac{1}{3}} 81$       (v)  $\log_3 9^k$

(c) State the value of

(i)  $\log_{625} 625$       (ii)  $\log_{25} 625$       (iii)  $\log_{\frac{1}{25}} 625$       (iv)  $\log_{\sqrt{5}} 625$       (v)  $\log_5 625^k$

(d) State the value of

(i)  $\log_{10000} 10000$       (ii)  $\log_{100} 10000$       (iii)  $\log 10000$

(iv)  $\log_{\frac{1}{100}} 10000$       (v)  $\log 10000^k$

6. (a) Given that  $\log_4 a = x$

- (i) express the relationship between  $a$  and  $x$  using powers;
- (ii) write down  $\log_2 a$  in terms of  $x$ ;
- (iii) write down  $\log_{16} a$  in terms of  $x$ .

(b) Given that  $\log_9 b = y$

- (i) express the relationship between  $b$  and  $y$  using powers;
- (ii) write down  $\log_3 b$  in terms of  $y$ ;
- (iii) write down  $\log_{\frac{1}{3}} a$  in terms of  $y$ .

(c) Given that  $\log_{25} c = z$

- (i) express the relationship between  $c$  and  $z$  using powers;
- (ii) write down  $\log_{\frac{1}{25}} c$  in terms of  $z$ ;
- (iii) write down  $\log_{\sqrt{125}} c$  in terms of  $z$ .

## Calculations with Logarithms – Essential Review

### Answers

1. (a) (i) 2 (ii) 5 (iii)  $-1$  (iv) 0 (v)  $-4$   
 (b) (i) 2 (ii)  $-2$  (iii)  $-4$  (iv) 0 (v) not possible  
 (c) (i)  $-1$  (ii)  $-2$  (iii) 1 (iv) 0 (v) 7  
 (d) (i) 2 (ii) 4 (iii) 3 (iv)  $-3$  (v) 0
2. (a) (i)  $10^1$  (ii)  $10^2$  (iii)  $10^{-1}$  (iv)  $10^0$  (v)  $10^{\frac{5}{2}}$   
 (b) (i)  $2^4$  (ii)  $2^{-4}$  (iii)  $2^0$  (iv)  $2^{-\frac{1}{2}}$  (v) not possible  
 (c) (i)  $3^1$  (ii)  $3^2$  (iii)  $3^{-2}$  (iv)  $3^{-\frac{3}{2}}$  (v) not possible  
 (d) (i)  $(\sqrt{2})^2$  (ii)  $(\sqrt{2})^4$  (iii)  $(\sqrt{2})^{\frac{9}{2}}$  (iv)  $(\sqrt{2})^{-2}$  (v)  $(\sqrt{2})^{-7}$
3. (a) (i) 10 (ii) 100 (iii)  $\frac{1}{10}$  (iv) 1 (v)  $\sqrt{10}$   
 (b) (i) 4 (ii) 8 (iii)  $\frac{1}{16}$  (iv)  $\frac{1}{\sqrt{2}}$  (v) 1  
 (c) (i)  $\frac{1}{4}$  (ii)  $\frac{1}{2}$  (iii) 1 (iv) 2 (v) 32  
 (d) (i)  $\sqrt{2}$  (ii)  $\frac{1}{2}$  (iii)  $2\sqrt{2}$  (iv) 1 (v)  $\sqrt[4]{2}$
4. (a) (i) 1 (ii) 2 (iii) 17 (iv)  $\frac{1}{2}$  (v) 0  
 (b) (i) 3 (ii) 4 (iii)  $\frac{1}{2}$  (iv)  $\frac{3}{2}$  (v) not possible  
 (c) (i)  $-1$  (ii)  $-2$  (iii) 1 (iv) 0 (v)  $\frac{1}{2}$   
 (d) (i) 1 (ii) 2 (iii) 3 (iv)  $-3$  (v) not possible
5. (a) (i) 2 (ii) 3 (iii) 6 (iv) 12 (v)  $3k$   
 (b) (i) 1 (ii) 2 (iii) 4 (iv)  $-4$  (v)  $2k$   
 (c) (i) 1 (ii) 2 (iii)  $-2$  (iv) 8 (v)  $4k$   
 (d) (i) 1 (ii) 2 (iii) 4 (iv)  $-2$  (v)  $5k$
6. (a) (i)  $4^x = a$  (ii)  $2x$  (iii)  $\frac{x}{2}$   
 (b) (i)  $9^y = b$  (ii)  $2y$  (iii)  $-2y$   
 (c) (i)  $25^z = c$  (ii)  $-z$  (iii)  $\frac{4z}{3}$