**IB Maths SL Unit 2 Chapter 2 Practice Problems – FUNCTIONS Markscheme**

**Remember that the methods shown are not the only valid possible options! Note the command terms and the associated number of points. For example, WRITE DOWN usually only results in 1 point, whereas FIND would give more points (reasoning + answer)**

**1.** (a) (i) $\sqrt{6}$ A1 N1

(ii) 9 A1 N1

(iii) 0 A1 N1

(b) $x < 5$ A2 N2

(c) $(g \circ f)(x) = (\sqrt{x-5})^2$ (M1)
$= x - 5$ A1 N2

**[7]**

**2.** (a) **METHOD 1**

$f(3) = \sqrt{7}$ (A1)

$(g \circ f)(3) = 7$ A1 N2

**METHOD 2**

$(g \circ f)(x) = \sqrt{x+4}^2$ $(= x + 4)$ (A1)

$(g \circ f)(3) = 7$ A1 N2

(b) For interchanging $x$ and $y$ (seen anywhere) (M1)
Evidence of correct manipulation A1

*eg* $x = \sqrt{y+4}, x^2 = y+4$

$f^{-1}(x) = x^2 - 4$ A1 N2

(c) $x \geq 0$ A1 N1

**[6]**

**3.** (a) attempt to form composite (M1)

*e.g.* $g(7-2x), 7-2x+3$

$(g \circ f)(x) = 10 - 2x$ A1 N2 2

(b) $g^{-1}(x) = x - 3$ A1 N1 1

(c) **METHOD 1**

valid approach (M1)

*e.g.* $g^{-1}(5)$, 2, $f(5)$

$f(2) = 3$ A1 N2 2

**METHOD 2**

attempt to form composite of $f$ and $g^{-1}$ (M1)

*e.g.* $(f \circ g^{-1})(x) = 7 - 2(x - 3)$, $13 - 2x$

$(f \circ g^{-1})(5) = 3$ A1 N2 2

**[5]**

**4.** (a) attempt to form composite (M1)

*e.g.* $f(2x - 5)$

$h(x) = 6x - 15$ A1 N2 2

(b) interchanging $x$ and $y$ (M1)

evidence of correct manipulation (A1)

*e.g.* $y + 15 - 6x$, $\frac{x}{6} = y - \frac{5}{2}$

$h^{-1}(x) = \frac{x+15}{6}$ A1 N3 3

**[5]**

**5.** (a) attempt to form composition (in any order) (M1)

$(f \circ g)(x) = (x - 1)^2 + 4$ $(x^2 - 2x + 5)$ A1 N2

(b) **METHOD 1**

vertex of $f \circ g$ at (1, 4) (A1)

evidence of appropriate approach (M1)

*e.g.* adding $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to the coordinates of the vertex of $f \circ g$

vertex of $h$ at (4, 3) A1 N3

**METHOD 2**

attempt to find $h(x)$ (M1)
*e.g.* $((x-3)-1)^2 + 4 - 1$, $h(x) = (f \circ g)(x-3) - 1$

$h(x) = (x-4)^2 + 3$ (A1)

vertex of $h$ at (4, 3) A1 N3

(c) evidence of appropriate approach (M1)
*e.g.* $(x-4)^2 + 3$, $(x-3)^2 - 2(x-3) + 5 - 1$

simplifying A1
*e.g.* $h(x) = x^2 - 8x + 16 + 3$, $x^2 - 6x + 9 - 2x + 6 + 4$
$h(x) = x^2 - 8x + 19$ AG N0

(d) **METHOD 1**

equating functions to find intersection point (M1)
*e.g.* $x^2 - 8x + 19 = 2x - 6$, $y = h(x)$

$x^2 - 10x + 25 = 0$ A1

evidence of appropriate approach to solve (M1)
*e.g.* factorizing, quadratic formula

appropriate working A1
*e.g.* $(x-5)^2 = 0$

$x = 5$ $(p = 5)$ A1 N3

**METHOD 2**

attempt to find $h'(x)$ (M1)
$h'(x) = 2x - 8$ A1

recognizing that the gradient of the tangent is the derivative (M1)
*e.g.* gradient at $p = 2$

$2x - 8 = 2$ $(2x = 10)$ A1

$x = 5$ A1 N3

**[12]**

**6.** (a) for interchanging $x$ and $y$ (may be done later) (M1)
*e.g.* $x = 2y - 3$

$g^{-1}(x) = \frac{x+3}{2}$ $\left(\text{accept } y = \frac{x+3}{2}, \frac{x+3}{2}\right)$ A1 N2

(b) **METHOD 1**

$g(4) = 5$ (A1)

evidence of composition of functions (M1)

$f(5) = 25$ A1 N3

**METHOD 2**

$f \circ g(x) = (2x - 3)^2$ (M1)

$f \circ g(4) = (2 \times 4 - 3)^2$ (A1)

$= 25$ A1 N3

**[5]**

**7.** (a) $(f \circ g): x \mapsto 3(x + 2)$ $(= 3x + 6)$ A2 N2

(b) **METHOD 1**

Evidence of finding inverse functions M1

*e.g.* $f^{-1}(x) = \frac{x}{3}$ $g^{-1}(x) = x - 2$

$f^{-1}(18) = \frac{18}{3} (= 6)$ (A1)

$g^{-1}(18) = 18 - 2\ (= 16)$ (A1)

$f^{-1}(18) + g^{-1}(18) = 6 + 16 = 22$ A1 N3

**METHOD 2**

Evidence of solving equations M1

*e.g.* $3x = 18$, $x + 2 = 18$

$x = 6$, $x = 16$ (A1)(A1)

$f^{-1}(18) + g^{-1}(18) = 6 + 16 = 22$ A1 N3

**[6]**

**8.** (a) **METHOD 1**

For $f(-2) = -12$ (A1)

$(g \circ f)(-2) = g(-12) = -24$ A1 N2

**METHOD 2**

$(g \circ f)(x) = 2x^3 - 8$ (A1)

$(g \circ f)(-2) = -24$ A1 N2

(b) Interchanging $x$ and $y$ (may be done later) (M1)

$x = y^3 - 4$ A1

$f^{-1}(x) = \sqrt[3]{(x+4)}$ A2 N3

**[6]**

**9.** (a) Evidence of attempting to form composition (M1)

Correct substitution $(h \circ g)(x) = \dfrac{5(3x-2)}{(3x-2)-4}$ A1

$= \dfrac{5(3x-2)}{(3x-6)} \quad \left(= \dfrac{15x-10}{3x-6}\right) \left(= \dfrac{5(3x-2)}{3(x-2)}\right)$ A1 N2

(b) Evidence of using numerator = 0 (M1)

*eg* $15x - 10 = 0$ $(3x - 2 = 0)$

$x = \dfrac{2}{3} \ (= 0.667)$ A2 N3

**[6]**

10. (a) (i) $p = 2$ A1 N1

(ii) $q = 1$ A1 N1

(b) To solve these, set y = 0 to find the x-intercept, then solve for x. You get two answers, but only x = 2 works with the equation. Then set x = 0 and plug it in to find the y – intercept, which conveniently comes out to be 2 (the rational part becomes zero).

We'll review this problem as a class if there are concerns.