

Solution Key

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Date: _____

IB Math SL Sequences and Series

I will use the following
 $U_{n+1} = U_1 r^n$, For the
other method, please
contact me.

- $U_1 = 3700$ 1. A painting was purchased for \$3700. The painting increases by 15% in value every year.
a) Write the general term for the value of the painting.
b) Find the value of the painting after 8 years.
c) Find the value of the painting after 20 years.

$$r = \frac{115}{100} = 1.15$$

$$\textcircled{a} U_{n+1} = (3700)(1.15)^n$$

$$\text{or } U_n = 3700(1.15)^{n-1}$$

$$\textcircled{c} \text{ After 20 yrs}$$

$$U_{20+1} = 3700(1.15)^{20}$$

$$= \$60556$$

$$\textcircled{b} \text{ After 8 years!}$$

$$U_{8+1} = 3700(1.15)^8 = \$11318.38$$

2. A car was bought for \$34,500. The car value depreciates by 8% every year.

$$U_1 = 34500$$

- a) Write the general term for the value of the car.
b) Find the value of the car after two years.

$$\text{Now} = 100\%$$

$$\text{later} = 92\%$$

$$r = \frac{92}{100} = 0.92$$

- c) Find the value of the car after 15 years.

$$\textcircled{a} U_{n+1} = 34500(0.92)^n$$

$$\text{or } U_n = 34500(0.92)^{n-1}$$

$$\textcircled{c} U_{15+1} = 34500(0.92)^{15}$$

$$= 9877$$

$$\textcircled{b} U_{2+1} = 34500(0.92)^2$$

$$= \$29200$$

3. A well initially produces 48 000L of water. The production of the well decreases by 7% every year.

$$U_1 = 48000L$$

- a) How many litres of water will be produced in the 5th year?
b) How many total litres will be produced in the first 16 years?
c) How many litres will be produced until the well runs dry?

$$\text{Now} = 100\%$$

$$\text{later} = 93\%$$

$$r = \frac{93}{100} = 0.93$$

$$\textcircled{a} U_{5+1} = 48000(0.93)^5$$

production in the 1st yr = 48000 = U_1
production in the 2nd yr = 48000 x 93% = U_2
3rd yr = 48000 x (93%)² = U_3

$$\textcircled{a} U_1 = 48000$$

$$U_5 = ?$$

$$r = 0.93$$

$$n = 5$$

$$\therefore U_5 = U_1 r^{5-1}$$

$$= 48000(0.93)^4$$

$$= 35906 \text{ Liters}$$

$$\therefore U_1 = 48000$$

$$\text{after in the } 5^{\text{th}} \text{ yr} = U_5$$

- Answers
- | | | |
|---------------------------------|----------------|----------------|
| 1) a) $U_n = 3700(1.15)^{n-1}$ | b) \$11 318.38 | c) \$60 556.19 |
| 2) a) $U_n = 34500(0.92)^{n-1}$ | b) \$29 200.80 | c) \$9877.26 |
| 3) a) 35 906.50 | b) 470 995.34 | c) 685 714.29 |

$$\textcircled{b} n = 16, S_{16} = ?, r = 0.93, U_1 = 48000$$

$$S_{16} = \frac{48000(1 - 0.93^{16})}{1 - 0.93} = 470995 \quad \textcircled{c} \text{ Find } S_{\infty} \quad \textcircled{smiley}$$

Ms. Raafa Abdulla

Solu Key

Quiz

Name: Raafat H. Dulla date: _____

- 1) Determine the position of the given term to complete the following statement.

170 is the ----- th term of -4, 2, 8, ... find "n"

$$\begin{array}{l|l} d = 6 & U_n = U_1 + (n-1)d \\ U_1 = -4 & 170 = -4 + (n-1)(6) \\ U_n = 170 & n = 30 \end{array}$$

- 2) For the arithmetic series, determine the value of n.

$$u_1 = 8, u_n = 68, S_n = 608$$

$$S_n = \frac{n}{2} (u_1 + u_n)$$

$$608 = \frac{n}{2} (8 + 68)$$

$$n = 16$$

- 3) Determine a formula for the nth term of each geometric sequence.

$$r = 2, u_1 = 3$$
~~$$U_n = 3(r^{n-1})$$~~

$$U_n = 3 \cdot 2^{n-1}$$

- 4) What is the value of the first term for each geometric series described?

Express your answers to the nearest tenth, if necessary.

a) $S_n = 33, u_n = 48, r = -2$

$$S_n = \frac{U_1 (1 - (-2)^n)}{1 - (-2)} = 33$$

$$= U_1 - \frac{U_1 (-2)^n}{-2} = 99$$

$$\Rightarrow U_1 - (-96) = 99 \Rightarrow U_1 = 3$$

- 5) Given the infinite geometric series:

$7 - 2.8 + 1.12 - 0.448 + \dots$ What is the sum of the series?

$$r = \frac{-2.8}{7} = -0.4 \Rightarrow |r| < 1 \Rightarrow \text{convergent} \Rightarrow S_\infty \text{ can be used}$$

$$U_1 = 7$$

$$S_\infty = \frac{U_1}{1-r} = \frac{7}{1-(-0.4)} = 5$$

Compound Interest

9- [a] $U_1 = \$3000$ $r = 1.1$ $n = 3$

$$U_{n+1} = U_1 r^n$$
$$U_4 = \$3000 (1.1)^3$$
$$= 3993$$

[b] Interest = $3993 - 3000 = 993$

[10] Same as question 9

[11] Same as question 9

[12] $U_{n+1} = U_1 r^n$ $r = 1.09 = \frac{109\%}{100}$

$$= 80000 (1.09)^3$$
$$= 103602$$

Interest = $103602 - 80000 = \$23602$

[13] $U_1 = 100000$

$n = 10 = 5 \times 2$

$r \Rightarrow \frac{8\%}{2} = 4\% \Rightarrow \frac{104\%}{100\%} = 1.04\%$

$$U_{10} = U_1 r^n$$
$$= 100000 (1.04)^{10} = 148024$$

[14] $U_1 = \$45000$

4 quarters/year

$n = \frac{21 \text{ months}}{12 \text{ months/yr}} \Rightarrow \frac{7}{4} \text{ years} \times 4 = \boxed{7}$

$r \Rightarrow \frac{7.5}{4} = 1.875\% \Rightarrow r = \frac{1.01875}{100} = \boxed{1.01875}$

$$U_7 = 45000 \times (1.01875)^7 = \boxed{51249}$$