Functions - November 28, 2016 *[41 marks]*

1a. Let $f(x) = 3x - 2$ and $g(x) = \frac{5}{3x}$, for $x \neq 0$. *[2 marks]*

Find $f^{-1}(x)$.

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1b. Show that $\left(g \circ f^{-1}\right)(x) = \frac{5}{x+2}$. *[2 marks]*

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1c. Let $h(x) = \frac{5}{x+2}$, for $x \geqslant 0$. The graph of h has a horizontal asymptote at $y = 0$. *[2 marks]*

Find the y-intercept of the graph of h.

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1d. Hence, sketch the graph of h. *[3 marks]*

1e. For the graph of h^{-1}, write down the x-intercept; *[1 mark]*

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1f. For the graph of h^{-1}, write down the equation of the vertical asymptote. *[1 mark]*

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1g. Given that $h^{-1}(a) = 3$, find the value of a. *[3 marks]*

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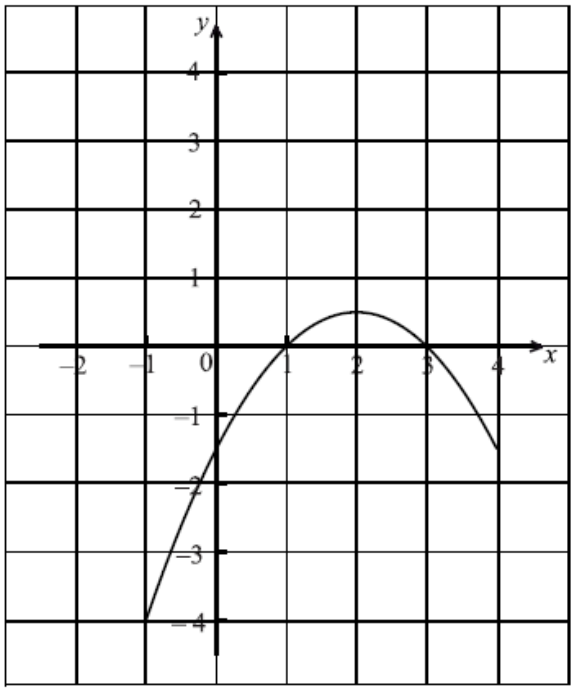
2a. Part of the graph of a function f is shown in the diagram below. *[2 marks]*



On the same diagram sketch the graph of $y = -f(x)$.

2b. Let $g(x) = f(x + 3)$. *[4 marks]*

(i) Find $g(-3)$.

(ii) Describe fully the transformation that maps the graph of f to the graph of g.

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3a. Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots. *[5 marks]*

Find the value of k.

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3b. The line $y = p$ intersects the graph of f. Find all possible values of p. *[2 marks]*

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4a. Let $f(x) = x^2$ and $g(x) = 2(x - 1)^2$. *[2 marks]*

The graph of g can be obtained from the graph of f using two transformations.

Give a full geometric description of each of the two transformations.

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4b. The graph of g is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ to give the graph of h. *[4 marks]*

The point $(-1, 1)$ on the graph of f is translated to the point P on the graph of h.

Find the coordinates of P.

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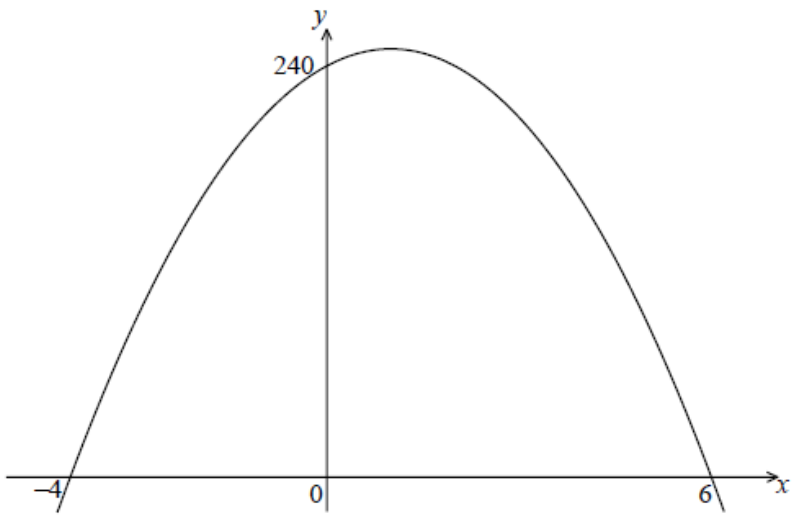
5a. The following diagram shows part of the graph of a quadratic function f. *[2 marks]*



The x-intercepts are at $(-4, 0)$ and $(6, 0)$, and the y-intercept is at $(0, 240)$.

Write down $f(x)$ in the form $f(x) = -10(x - p)(x - q)$.

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5b. Find another expression for $f(x)$ in the form $f(x) = -10(x - h)^2 + k$. *[4 marks]*

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5c. Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$. *[2 marks]*

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