GOOOOOOOOOOOOOOOOOOD Luck!

Boundries

THE UNIVERSAL AMERICAN SCHOOL
UAS
DUBAI

Mathematics Department

Grade 10 B

Raafa

Summative Assessment

March 1st 2017

Name: Answer Key

Instructions:

1. Answer all the questions in the spaces provided.
2. Show ALL the working out required to get answers.
3. A graphic display calculator is allowed for this paper.
4. Unless stated otherwise, give all answers exactly or rounded to 3 significant figures.
5. The total marks for this paper is 60.

Common Core Standards:

1. HSF-BF.A.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
2. HSF-LE.A.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. HSF-LE.A.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
4. HSF-BF.B.5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
5. HSN-Q.A.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

1. Solve $\log_2 x + \log_2(x-2) = 3$

① $\log_2 x(x-2) = 3$

① $x^2 - 2x = 2^3$

① $x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4 \quad x \neq -2$

①

take off one mark if they kept $x=-2$

(Total 4 marks)

2. Given that $\log_5 x = y$, express each of the following in terms of y.

(a) $\log_5 x^2$ = ?

(b) $\log_5 \frac{1}{x}$

(c) $\log_{25} x$ [hint: change the log base into 5]

[a] $\log_5 x^2 = 2\log_5 x = 2(y) = 2y$

① ①

[b] $\log_5 \frac{1}{x} = \log_5 x^{-1} = -\log_5 x = -(y) = -y$

① or ①

[c] $\log_{25} x = \frac{\log_5 x}{\log_5 25} = \frac{y}{\log_5 5^2} = \frac{y}{2}$

① ①

(Total 6 marks)

3. Let $a = \log x$, $b = \log y$, and $c = \log z$.

Write $\log\left(\frac{x^2\sqrt{y}}{z^3}\right)$ in terms of a, b and c.

$\log x^2 + \log \sqrt{y} - \log z^3$ (1) (1)

$2\log x + \frac{1}{2}\log y - 3\log z$ (1) (1) (1)

$2(a) + \frac{1}{2}(b) - 3(c)$ (1)

$= 2a + \frac{1}{2}b - 3c$

(Total 6 marks)

4. Find the **exact** solution of the equation $9^{2x} = 27^{(1-x)}$.

$3^{2(2x)} = 3^{3(1-x)}$ (1) (1)

$2(2x) = 3(1-x)$ (1)

$4x = 3 - 3x$

$7x = 3$

$x = \frac{3}{7}$ (1)

(Total 4 marks)

5. The mass M grams, of radioactive substance remaining after t years is given by $M = 1000 \times e^{-0.04t}$ grams. Find the time (to the nearest year) taken for the mass to reach 25 grams.

$25 = 1000\, e^{-0.04t}$ (1)

$\frac{25}{1000} = e^{-0.04t}$ (1)

$\ln \frac{25}{1000} = -0.04t$ (1)

$t = \frac{\ln 25/1000}{-0.04} = 92$ years (1) (1)

(Total 5 marks)

6. For the arithmetic sequence 2, 7, 12, 17, ... find the following:

(a) U_n

$d = 5$ ①

$U_1 = 2$ ①

$U_n = 2 + (n-1)5$ ①

(b) U_{18}

$U_{18} = 2 + (18-1)5$ ①

$= 2 + (17)5$

$= 2 + 85 = 87$ ①

(Total 4 Marks)

7. Consider the arithmetic sequence 3, 9, 15, ..., 1353.

(a) Write down the common difference.

$d = 9 - 3 = 6$ ①

(b) Find the number of terms in the sequence.

① $U_1 = 3$, $d = 6$

① $U_n = 1353$

$U_n = U_1 + (n-1)d$

① $1353 = 3 + (n-1)6$

$\frac{1350}{6} = \frac{6(n-1)}{6}$

$225 = n - 1$

$226 = n$ ①

(Total 4 marks)

8. Arturo goes swimming every week. He swims 200 metres in the first week. Each week he swims 30 metres more than the previous week. He continues for one year (52 weeks).

(a) How far does Arturo swim in the final week (52nd week)?

$U_1 = 200$ (1)
$n = 52$
$d = 30$

$U_n = U_1 + (n-1)d$

$U_{52} = 200 + (52-1)30$ (1)

$= 200 + 51 \times 30$

$U_{52} = 1730$ (1)

(Total 3 marks)

9. Peter began a workout program. On the first day of training, he will do 7 push-ups. On the second day, he will do 13. On the third day, he will do 19. Assuming Peter is able to keep the same increase from day to day,

a) On which day will he do 277 push-ups?

$U_1 = 7$
$U_2 = 13$] 6
$U_3 = 19$] 6

$d = 6 = 13 - 7 = 19 - 13 = 6$

$*U_n = 277$ $*n?$

$U_n = U_1 + (n-1)d$ (1)

$277 = 7 + (n-1)6$ (1) (1)

$\frac{270}{6} = n - 1$

$45 + 1 = n$

$46 = n$ (1)

On ~~After~~ 46th days

(Total 4 Marks)

10. A museum purchases a painting for $ 15 000. The painting increases in value each year by 10% of the original price. What is the value of the painting **at the end** of the 10th year?

$U_{n+1} = U_1 r^n$ (1) ✓

$U_{11} = 15000(1.10)^{10}$

$\$ 38906.14$ (1)

$n = 10$ (1)

$U_1 = 15000$

$r = \frac{110}{100} = 1.10$ (1)

✓

(Total 4 Marks)

11. In a geometric sequence, $U_3 = 45$ and $U_5 = 1125$. Find the **common ratio**, r, and the **first term**.

$U_5 = U_3 \times r^2$

$1125 = 45(r^2)$ (1)

$r = 5$ (1)

$U_1 = \frac{U_3}{r^2} = \frac{45}{25}$ (1)

$U_1 = 1.8$ (1)

$\frac{U_5}{1125}$ — $\frac{U_3}{45}$ —

(Total 4 Marks)

12. An arithmetic sequence has consecutive terms $(7x + 4)$, $(5x + 17)$ and $(13x + 3)$. Determine the value of x.

$5x+17-(7x+4) = 13x+3-(5x+17)$ (1)

$5x+17-7x-4 = 13x+3-5x-17$ (1)

$-2x+13 = 8x-14$ (1)

$+2x \quad +14 \quad +2x \quad +14$

$27 = 10x$

$2.7 = x$ (1)

(Total 4 Marks)

13. Let $f(x) = 3x$, $g(x) = 2x - 5$ and $h(x) = (f \circ g)(x)$.

(a) Find $h(x)$.

$h(x) = 3(2x-5) = 6x-15$ (1) (1)

(b) Find $h^{-1}(x)$

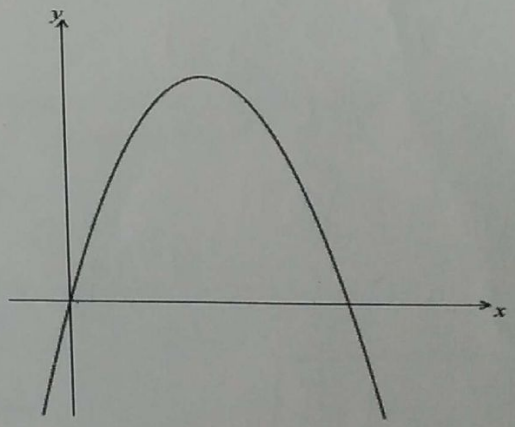
$y = 6x - 15$

$h^{-1}(x) \Rightarrow x = 6y - 15$ (1)

$\frac{x+15}{6} = y$ (1)

(Total 4 marks)

14. Let $f(x) = 8x - 2x^2$. Part of the graph of f is shown below.



(a) Find the x-intercepts of the graph.

$y = 0$ (1)

$8x - 2x^2 = 0$

$2x(4 - x) = 0$

$2x = 0$ | $4 - x = 0$

$x = 0$ | $x = 4$ (1)

(b) Write down the equation of the axis of symmetry.

$x = \frac{-b}{2a} = \frac{-8}{2(-2)} = \frac{-8}{-4} = 2.$

(1) (1)