IB Math SL - October 16, 2016 *[46 marks]*

1a. Find $\log_4 1024$. *[1 mark]*

$\log_4 1024$

$\log_4 4^5 = 5 \log_4 4 = 5$

1b. Given that $\log_2\left(\frac{32^x}{8^y}\right)$ can be written as $px + qy$, find the value of p and of q. *[4 marks]*

$\log_2 \frac{32^x}{8^y} = \log_2 32^x - \log_2 8^y$

$= x \log_2 2^5 - \log_2 2^{3y}$

$= 5x \log_2 2 - 3y \log_2 2$

$= 5x - 3y$

2a. Find the value of each of the following, giving your answer as an integer. *[2 marks]*

$\log_{36} 6$

$\log_{36} 6^1 = \log_{36} 36^{\frac{1}{2}}$

$= \frac{1}{2} \log_{36} 36 = \frac{1}{2}$

2b. $\log_6 4 + \log_6 9$ *[2 marks]*

$\log_6 4(9) = \log_6 36 = \log_6 6^2$

$= 2\log_6 6 = 2$

2c. $\log_6 2 - \log_6 12$ *[3 marks]*

$\log_6 \frac{2}{12} = \log_6 \frac{1}{6}$

$= \log_6 6^{-1}$

$= -1 \log_6 6$

$= -1$

3. Let $f(x) = 3\ln x$ and $g(x) = \ln 5x^3$. [4 marks]

Express $g(x)$ in the form $f(x) + \log a$, where $a \in \mathbb{Z}^+$.

$g(x) = \log 5x^3$

$= \log 5 + \log x^3$

$= \log 5 + 3\log x$

$= \log 5 + f(x)$

4a. Find the value of $\log_2 40 - \log_2 5$. *[3 marks]*

$\log_2 \frac{40}{5} = \log_2 8$

$= \log_2 2^3 = 3 \log_2 2$

$= 3$

4b. Find the value of $8^{\log_2 5}$. *[4 marks]*

$2^{3 \log_2 5} = 2^{\log_2 5^3} = 5^3 = 125$

5. Let $\log_3 p = 6$ and $\log_3 q = 7$.

[7 marks]

(a) Find $\log_3 p^2$.

(b) Find $\log_3 \left(\frac{p}{q}\right)$.

(c) Find $\log_3 (9p)$.

$\log_3 p^2 = 2 \log_3 p = 2 \times 6 = 12$

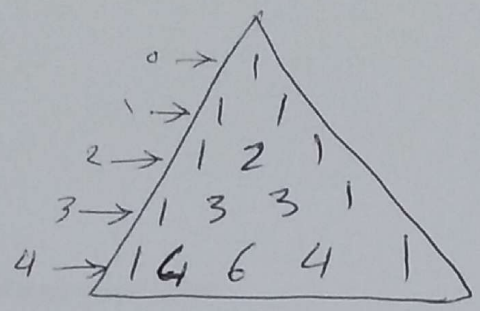
$\log \frac{p}{q} = \log p - \log q$

$= 6 - 7 = -1$

$\log_3 9p = \log_3 9 + \log_3 p$

$= \log_3 3^3 + \log_3 p$

$= 3 + 6 = 9$



6. Expand $(2-\frac{1}{x})^4$ and simplify your result. [3 marks]

$(2-\frac{1}{x})^4 = 1(2)^4(\frac{-1}{x})^0 + 4(2)^3(\frac{-1}{x})^1 + 6(2)^2(\frac{-1}{x})^2$
$+ 4(2)^1(\frac{-1}{x})^3 + 1(2)^0(\frac{-1}{x})^4$
$= 16 - \frac{32}{x} + \frac{24}{x^2} - \frac{8}{x^3} + \frac{1}{x^4}$

[7 marks]

7. Given that

$$\left(1+\frac{2}{3}x\right)^n(3+nx)^2 = 9+84x+\ldots,$$

Find the value of n

expand $(1+\frac{2}{3}x)^n$

$= 1 + n(\frac{2}{3}x)(1) + \ldots$

$= 1 + n(1)^{n-1}(\frac{2}{3}x)^1 + \ldots$

expand $(3+nx)^2$

$= 9 + 2(3)(nx) + (nx)^2$

from 1

$= 1 \cdot (2(3)(nx)) = 6nx$

+

from 2

$= (1)^{n-1}(\frac{2}{3}x)(9) = 6nx$

$\frac{12}{12}nx = \frac{84x}{12}$

$n = 7$

8. Given that $(3+\sqrt{7})^3 = p + q\sqrt{7}$ where p and q are integers, find [6 marks]

(a) p;

(b) q.

$1(3)^3(\sqrt{7})^0 + 3(3)^2(\sqrt{7})^1 + 3(3)^1(\sqrt{7})^2 + 1(3)^0(\sqrt{7})^3$

$27 + 27\sqrt{7} + 63 + 3\sqrt{7}$

$90 + 30\sqrt{7}$

p q