UNIVERSAL AMERICAN SCHOOL- DUBAI

GRADE 11 EXAMINATIONS

SEMESTER ONE, 2016-2017

Answer Key

Raafa

Mathematics SL- Paper 1

90 minutes

Grade / Level of Achievement: ____________

Teacher Name: ____________________________

Candidate Name: ____________________________

INSTRUCTIONS TO CANDIDATES

1. Read *all* your questions carefully before you attempt to answer them.
2. Answer in the space provided or as directed in each question.
3. There are 13 pages to this examination including the cover sheet.
4. The formula sheet is at the back of this exam booklet
5. No calculator is allowed

1. Solve $\log_2 x + \log_2 (x - 2) = 3$, for $x > 2$. *[7 marks]*

$\log_2 x(x-2) = 3$

$2^{\log_2 (x^2 - 2x)} = 2^3$

$x^2 - 2x = 8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4$ $\quad \cancel{x = -2}$

2a. Let $f(x) = x^2 + x - 6$. *[1 mark]*

Write down the y-intercept of the graph of f.

$y$int $= -6$

when $x = 0$ $\quad y = 0 + 0 - 6 = -6$

2b. Solve $f(x) = 0$. *[3 marks]*

$x^2 + x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ $\quad x = -2$

[3 marks]

3a. Write the expression $3\ln 2 - \ln 4$ in the form $\ln k$, where $k \in \mathbb{Z}$.

$3\ln 2 - \ln 4$

$\ln 2^3 - \ln 4$ A1

$\ln \frac{8}{4} = \ln 2$ A1 N2

[3 marks]

3b. Hence or otherwise, solve $3\ln 2 - \ln 4 = -\ln x$.

$\ln 2 = -\ln x$ 1

$e^{\ln 2} = e^{\ln x^{-1}}$ 1

$2 = \frac{1}{x}$ $\therefore x = \frac{1}{2}$ 1 N2

4a. Let $f(x) = px^2 + (10-p)x + \frac{5}{4}p - 5$.

Show that the discriminant of $f(x)$ is $100 - 4p^2$. [3 marks]

$\Delta = b^2 - 4ac$ 1

$= (10-P)^2 - \cancel{4}(P)(\frac{5}{\cancel{4}}P - 5)$ 1

$= 100 - 20P + P^2 - 5P^2 + 20P$ 1

$= 100 - 4P^2$ N0

4b. Find the values of p so that $f(x) = 0$ has two **equal** roots. [3 marks]

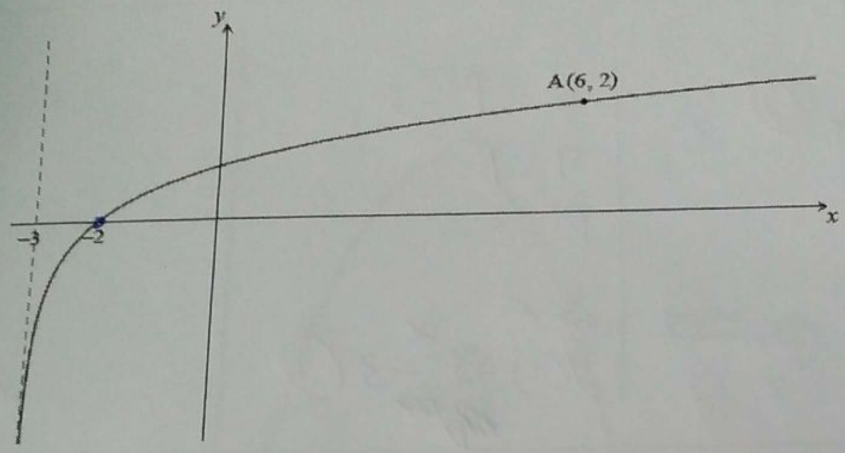
$\Delta = 100 - 4P^2 = 0$ 1

$(10-2P)(10+2P) = 0$ 1

$10 - 2P = 0$ $10 + 2P = 0$

$P = \pm 5$ A1 N2

5a. Let $f(x) = \log_p(x+3)$ for $x > -3$. Part of the graph of f is shown below. *[5 marks]*



The graph passes through A(6, 2), has an x-intercept at (−2, 0) and has an asymptote at $x = -3$.

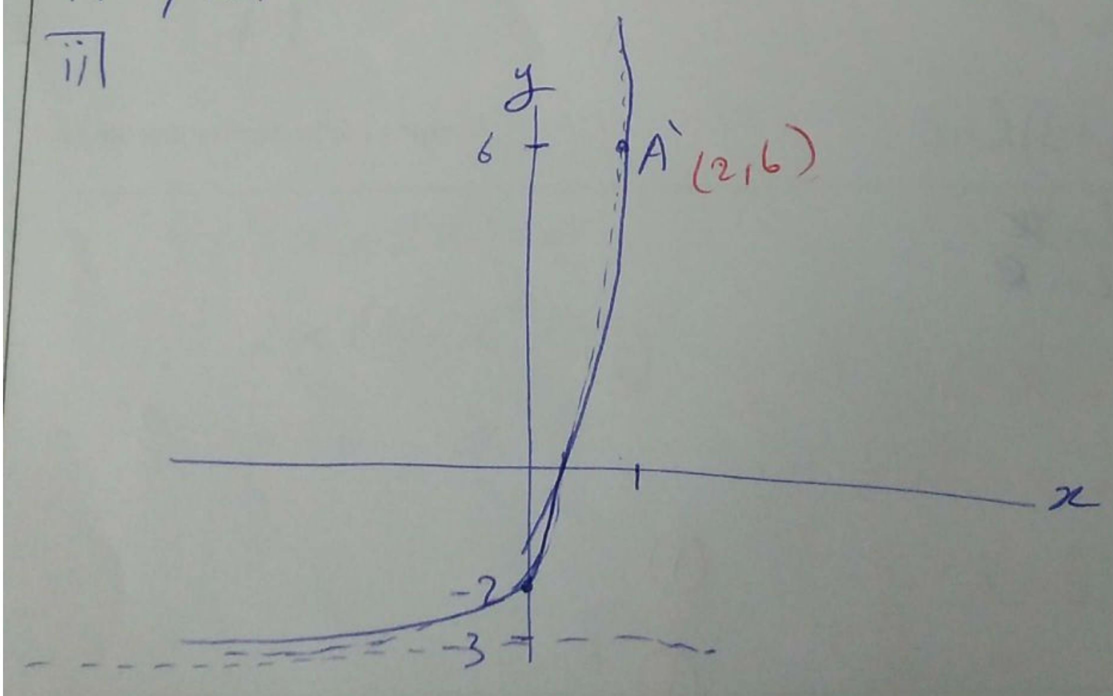
The graph of f is reflected in the line $y = x$ to give the graph of g.

(i) Write down the y-intercept of the graph of g.

(ii) Sketch the graph of g, noting clearly any asymptotes and the image of A.

i] y int = −2

ii]



5b. The graph of f is reflected in the line $y = x$ to give the graph of g. Find $g(x)$. *[4 marks]*

7a.

$g(x)$

$f(x) = \log_p x + 3$

$x = \log_p y + 3$ ②

$p^x = y + 3$ ①

$y = p^x - 3$

~~$(0, 2)$~~

$(2, 6)$

$6 = p^2 - 3$

$9 = p^2$

$p = 3$

~~$y =$~~

$g(x) = 3^x - 3$ ①

6. Let $f(x) = e^{x+3}$.

Solve the equation $f^{-1}(x) = \ln \frac{1}{x}$. *[4 marks]*

$f^{-1}(x)$:

$x = e^{y+3}$

$\ln x = \ln e^{y+3}$

$\ln x = (y+3) \ln e$

$y = \frac{\ln x}{\ln e} - 3$

$y = \ln x - 3 = \ln \frac{1}{x}$ ②

$\ln x - \ln \frac{1}{x} = 3$

$2 \ln x = \frac{3}{2}$ ①

$x = \sqrt{e^3}$ ① N2

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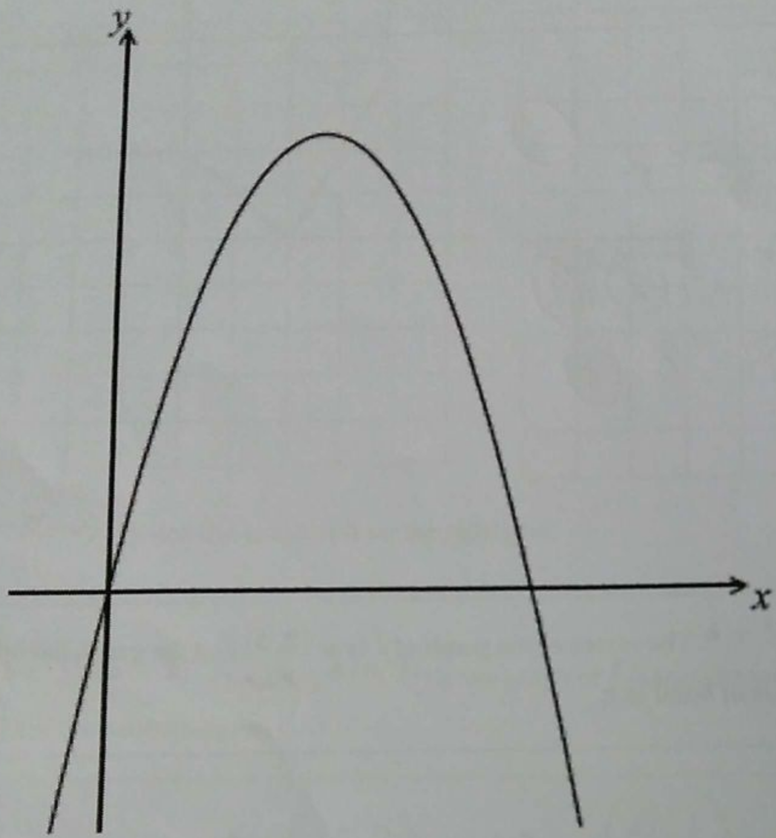
7a. Let $f(x) = 8x - 2x^2$. Part of the graph of *f* is shown below. *[4 marks]*



Find the *x*-intercepts of the graph.

$y = 8x - 2x^2 = 0$

$2x(4 - x^{\cancel{2}}) = 0$

$x = 0 \quad x = 4$

[3 marks]

7b. (i) Write down the equation of the axis of symmetry.

(ii) Find the y-coordinate of the vertex.

i) axis of sym.

$x = \frac{4+0}{2} = 2$ (1)

ii) $y = 8(2) - 2(2)^2$ (1)

$= 16 - 8 = 8$ (1)

8a. Let $f(x) = a(x-h)^2 + k$. The vertex of the graph of f is at $(2, 3)$ and the graph passes through $(1, 7)$. Write down the value of h and of k.

[2 marks]

$h = 2$ (1)

$k = 3$ (1)

[3 marks]

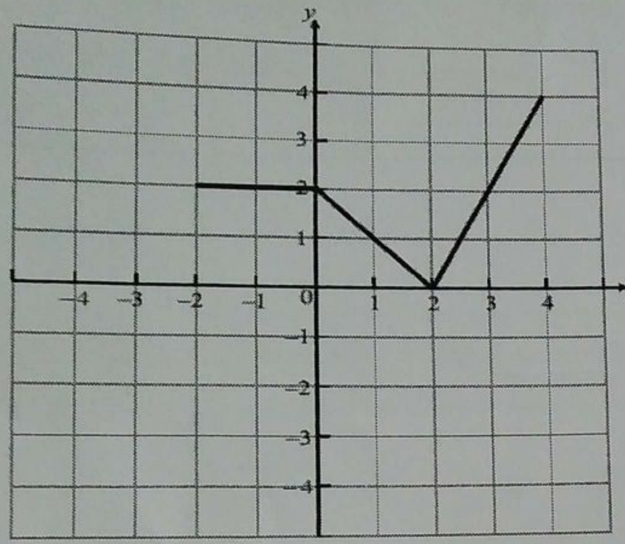
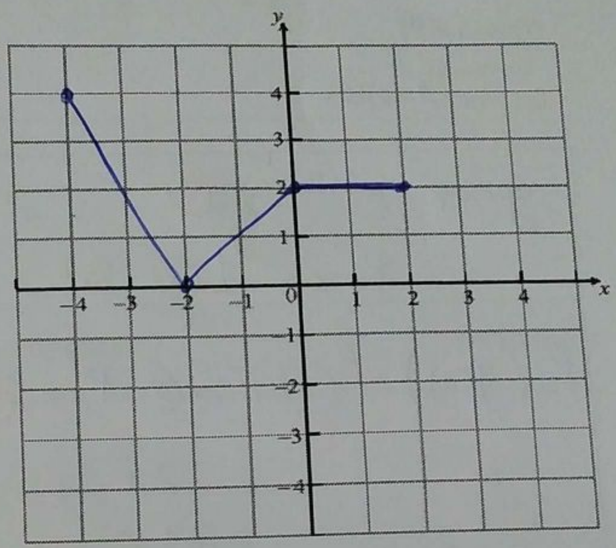
8b. Find the value of a.

$y = a(x-2)^2 + 3$ (1)

(1) $7 = a(1-2)^2 + 3$

$4 = a$ (1)

9a. The diagram below shows the graph of a function $f(x)$, for $-2 \le x \le 4$. *[3 marks]*

Let $h(x) = f(-x)$. Sketch the graph of h on the right grid.

9b. Let $g(x) = \frac{1}{2} f(x-1)$. The point $\mathbf{A(3, 2)}$ on the graph of f is transformed to the point P on the graph of g. Find the coordinates of P. *[3 marks]*

A: $(3,2) \xrightarrow[\text{right 1 unit}]{} (4,2) \xrightarrow[\text{compress vertically by } \frac{1}{2}]{} (4,1)$

10. Let $f(x) = 2x^3 + 3$ and $g(x) = e^{3x} - 2$. [5 marks]

(i) Find $g(0)$.

(ii) Find $(f \circ g)(0)$.

$g(0) = e^{3(0)} - 2$ ①

$= 1 - 2 = -1$ ① N2

$f(g(0)) = f(-1) = 2(-1)^3 + 3$ ① ①

$= -2 + 3$ ①

$= +1$ ← N3

11. Let $f(x) = k\log_2 x$.

Given that $f^{-1}(1) = 8$, find the value of k. [3 marks]

$y = K \log_2 x \qquad (8,1)$

$1 = K \log_2 8$ ①

$2^{1} = 2^{\log_2 8^K}$ ① → $8 = 2^3$

$2 = 8^K$

$2^1 = 2^{3K}$

$3K = 1$ ~~N1~~ A1

$K = \frac{1}{3}$ N2

$y = K \log_2 x$

$2^{y} = \cancel{2^{\log_2}} x^K$

$\cancel{2^y = x^K}$

$(2^x)^{\frac{1}{K}} = (y^K)^{\frac{1}{K}}$

$2^{\frac{x}{K}} = y$

$2^{\frac{1}{K}} = 8$

$2^{\frac{1}{K}} = 2^3$

$\frac{1}{K} = 3 \Rightarrow K = \frac{1}{3}$

12a. The fifth term in the expansion of the binomial $(a+b)^n$ is given by $\binom{10}{4} p^6 (2q)^4$.

Write down a and b, in terms of p and/or q. *[2 marks]*

$a = p$ ✓

$b = 2q$ ✓

12b. Write down an expression for the sixth term in the expansion. *[3 marks]*

$\binom{10}{5} p^5 (2q)^5 = 252\, p^5 (2q)^5$

(1) (1) (1)

$\frac{10!}{5!\,5!}$

$\frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5 \times 4 \times 3 \times 2 \times 5!} = 252$

13. In the expansion of $(3x+1)^n$, the coefficient of the term in x^2 is $135n$, where $n \in \mathbb{Z}^+$. Find n.

[7 marks]

(1) $\binom{n}{r}(3x)^{n-r}(1)^r = 135n\,x^2$

$\frac{n!}{r!\,(n-r)!}(3x)^{n-r} = 135n\,x^2$, $n-r=2$

(1) $n-r=2$

$r = n-2$

(1) $\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!} \times 2} \cdot 3^2 \cancel{x^2} = 135n\,\cancel{x^2}$

(1) $\frac{n^2-n}{2} \cdot 9 = 135n$

$n^2 - n = \frac{135 \times 2}{9} n$

(1) $n^2 - n = \frac{270}{9} n$

$n^2 - n = 30n$

$n^2 - 31n = 0$ (1)

$n(n-31) = 0$

~~$n=0$~~

(1) $n = 31$ ✓

N2

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14. Find the value of each of the following, giving your answer as an integer.

14a. $\log_6 36$ *[2 marks]*

$6^x = 36$
$6^x = 6^2$ ①
$x = 2$ ①

14b. $\log_6 4 + \log_6 9$ *[2 marks]*

$\log_6 4(9)$ ①
$\log_6 36 = \log_6 6^2 = 2$ ①

14c. $\log_6 2 - \log_6 12$ *[3 marks]*

$\log_6 \frac{2}{12}$ ①
$\log_6 6^{-1} = -1$ ①

15a. The first three terms of an infinite geometric sequence are 32, 16 and 8.

Write down the value of r. *[1 mark]*

$r = \frac{16}{32} = \frac{1}{2}$ ①

15b. Find u_6 .

[2 marks]

$U_6 = U_1 r^{n-1}$

$U_6 = 32\left(\frac{1}{2}\right)^{6-1}$ 1

$= \cancel{32}\left(\frac{1^5}{\cancel{2^5}}\right)$

$= 1$ 1

[2 marks]

15c. Find the sum to infinity of this sequence.

$S_\infty = \frac{U_1}{1-r}$

$= \frac{32}{1-\frac{1}{2}} = \frac{32}{1/2} = 64$

16a. In an arithmetic sequence, the first term is **2** and the second term is **5**.

Find the common difference. *[2 marks]*

$d = 5 - 2 = 3$

[2 marks]

16b. Find the eighth term.

$U_8 = U_1 + (n-1)d$

$= 2 + (8-1)(3)$

$= 2 + 7(3)$

$= 23$

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GRADE 11 EXAMINATIONS

SEMESTER ONE, 2016-2017

Answer Key

Mathematics SL- Paper 2

90 minutes

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1. Let $f(x) = kx^2 + kx$ and $g(x) = x - 0.8$. The graphs of f and g intersect at two distinct points.

Find the possible values of k. *[8 marks]*

$Kx^2 + Kx = (x - 0.8)$

$Kx^2 + Kx - x + 0.8$

$Kx^2 + (K-1)x + 0.8 = 0$

$a = K$ $\quad C = 0.8$

$b = K - 1$

$\Delta = b^2 - 4ac$

$(K-1)^2 - 4(K)(0.8)$

$= K^2 - 2K + 1 - 3.2K$

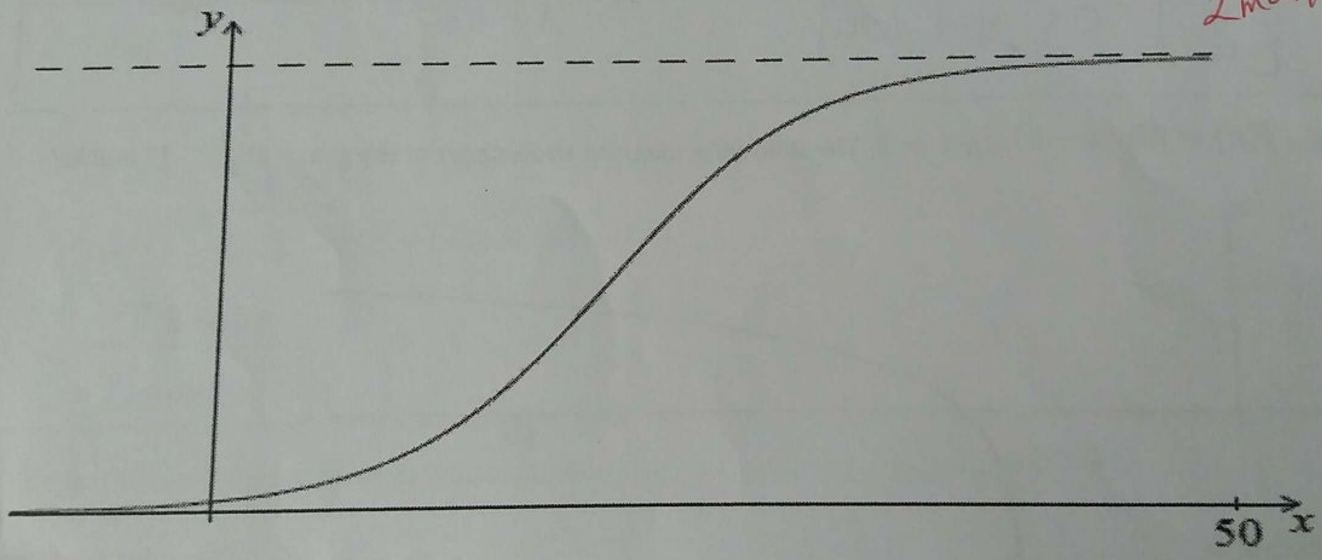
$= K^2 - 5.2K + 1 > 0$

$(K - 0.2)(K - 5) > 0$

~~$K > 0.2$~~ $\quad K > 5$

$K < 0.2$

2a. Let $f(x) = \frac{100}{(1+50e^{-0.2x})}$. Part of the graph of f is shown below. *~~[1 mark]~~* 2marks.



Write down $f(0)$.

$f(0) = \frac{100}{1+50e^{-0.2(0)}}$

$= 1.96$ or $\frac{100}{51}$

3b. Find

[2 marks]

2b. Solve $f(x) = 95$.

$95 = \frac{100}{1+50e^{-0.2(x)}}$

$-0.2x = \ln \frac{\frac{100}{95} - 1}{50}$

$x = \frac{\ln\left(\frac{100}{95} - 1 / 50\right)}{-0.2}$

$x = 34.3$

[3 marks]

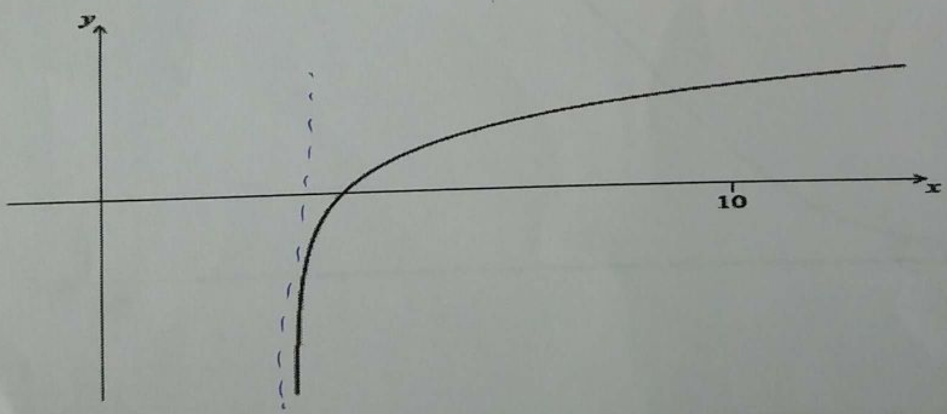
2c. Find the range of f.

Range.
$\{y \mid 0 < y < 100\}$

If $x \Rightarrow \infty$
$y = \frac{100}{1+50e^{-\infty}} = \frac{100}{1} = 100$

$\to 0$

3a. Let $f(x) = 2\ln(x-3)$, for $x > 3$. The following diagram shows part of the graph of f. [2 marks]



Find the equation of the vertical asymptote to the graph of f.

$x - 3 > 0$
$x > 3$
VA $\therefore$ $x = 3$

3b. Find the x-intercept of the graph of f. *[2 marks]*

$y = 0$ 1

$\frac{0}{2} = \frac{2}{2}\ln(x-3)$

$0 = \ln(x-3)$

$e^0 = x-3$

$1 = x-3$

$x = 4$ 1

4a. Let $f(x) = \frac{3x}{x-q}$, where $x \neq q$. *[2 marks]*

Write down the equations of the vertical and horizontal asymptotes of the graph of f.

VA:

$x - q = 0$

$\boxed{x = q}$ 1

HA: set $x = 1000000$

$y = \frac{3(1000000)}{(1000000) - q} = 3 \quad \therefore \boxed{y = 3}$ 1

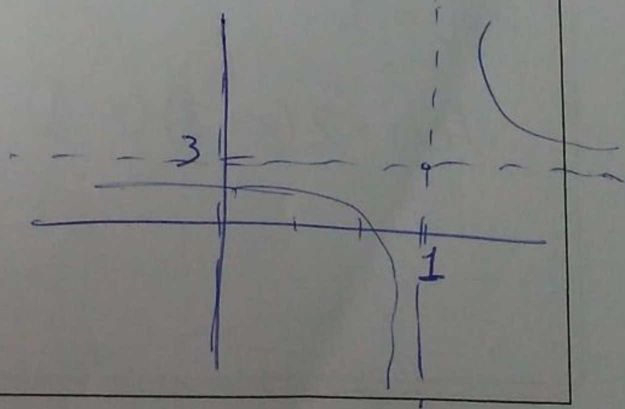
4b. The vertical and horizontal asymptotes to the graph of f intersect at the point $Q(1, 3)$.

Find the value of q. *[2 marks]*

$q = 1$ ✓

$3 = \frac{3}{1-q}$

$1 - q = 1$



5a. Let $f(x) = 2x^2 + 4x - 6$.

Express $f(x)$ in the form $f(x) = 2(x-h)^2 + k$. *[3 marks]*

$a = 2$

$b = 4$

$c = -6$

$h = \frac{-b}{2a} = \frac{-4}{2(2)} = -1$ 1

$k = 2(-1)^2 + 4(-1) - 6$

$= 2 - 4 - 6 = -8$

$y = 2(x+1)^2 - 8$ 1 1

5b. Write down the equation of the axis of symmetry of the graph of f. *[1 mark]*

ax. of sym $\Rightarrow x = \frac{-b}{2a} = \frac{-4}{2(2)} = -1$ 1

5c. Express $f(x)$ in the form $f(x) = 2(x-p)(x-q)$. *[2 marks]*

Xint. from the Calc.

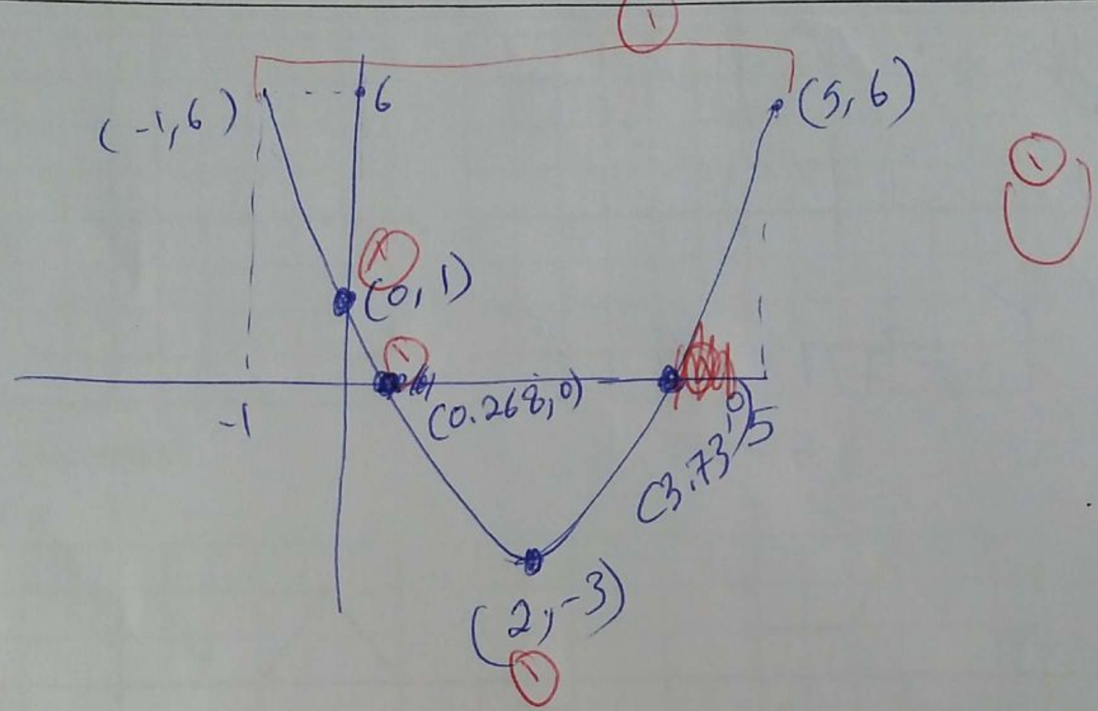
$p = -3$

$q = 1$

$\therefore f(x) = 2(x+3)(x-1)$

6a. Consider the function $f(x) = x^2 - 4x + 1$.

Sketch the graph of f, for $-1 \leq x \leq 5$. *[4 marks]*



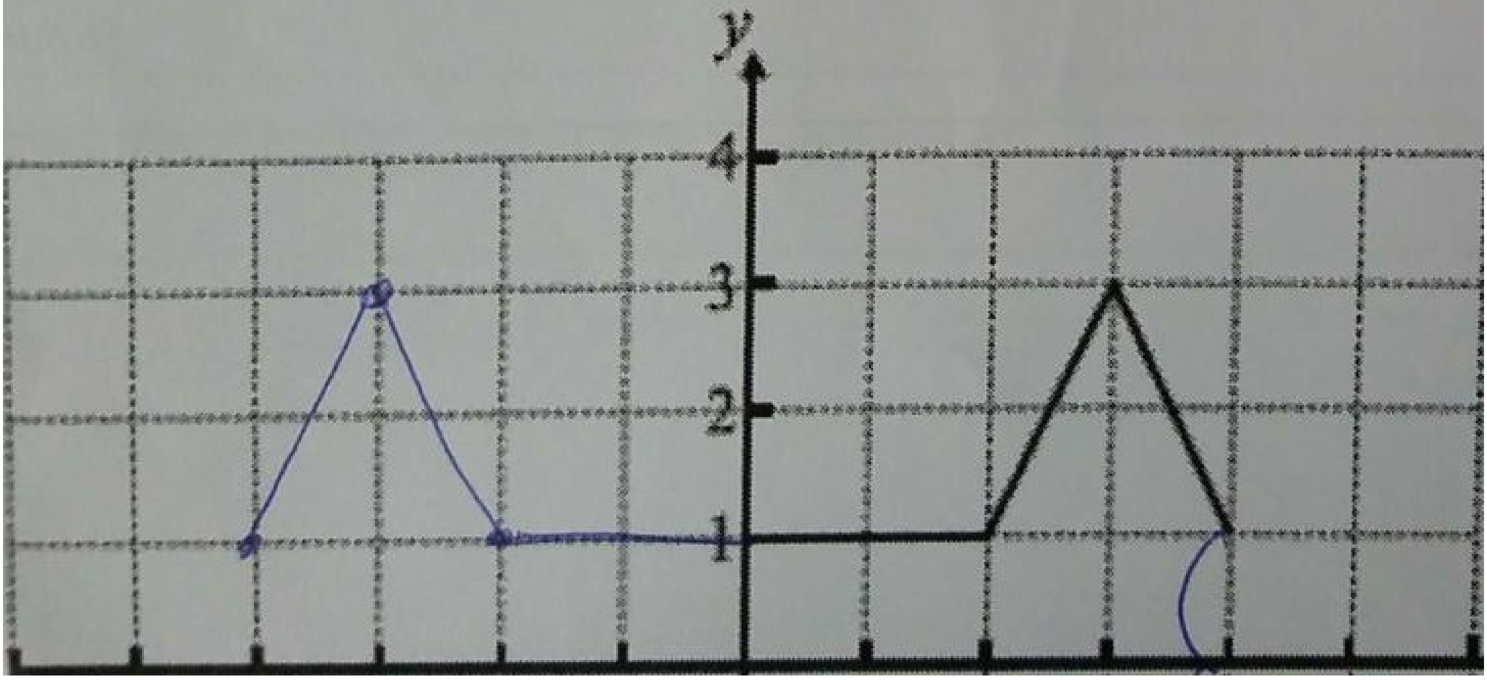
6b. This function can also be written as $f(x) = (x - p)^2 - 3$.

Write down the value of p. *[1 mark]*

$p = 2$

onsider the graph of f shown below.

e **same** grid sketch the graph of $y = f(-x)$.



7b. The following four diagrams show **images** of f under different transformations. *[2 marks]*

Diagram A

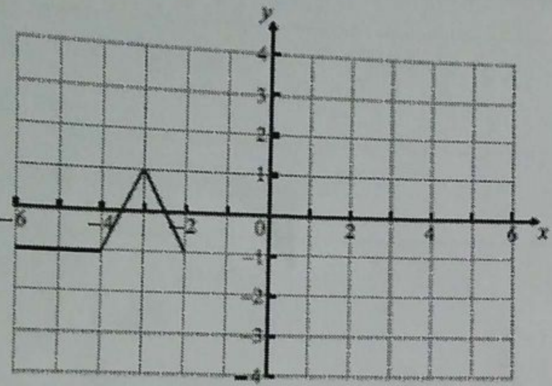


Diagram B

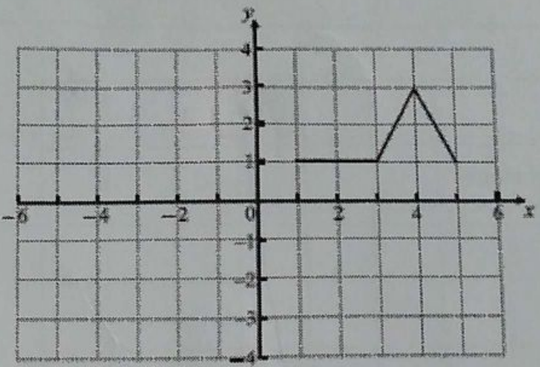


Diagram C

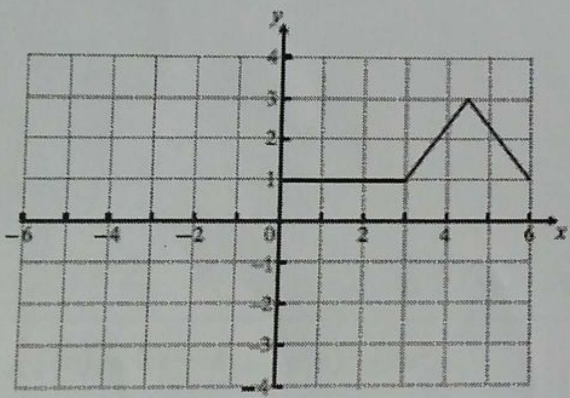
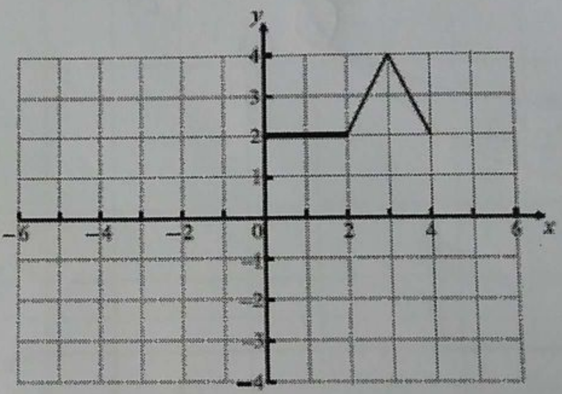


Diagram D



Complete the following table.

Description of transformation	Diagram letter
Horizontal stretch with scale factor 1.5	C
Maps f to $f(x)+1$	D

c. Give a full geometric description of the transformation that gives the image in Diagram A. *[2 marks]*

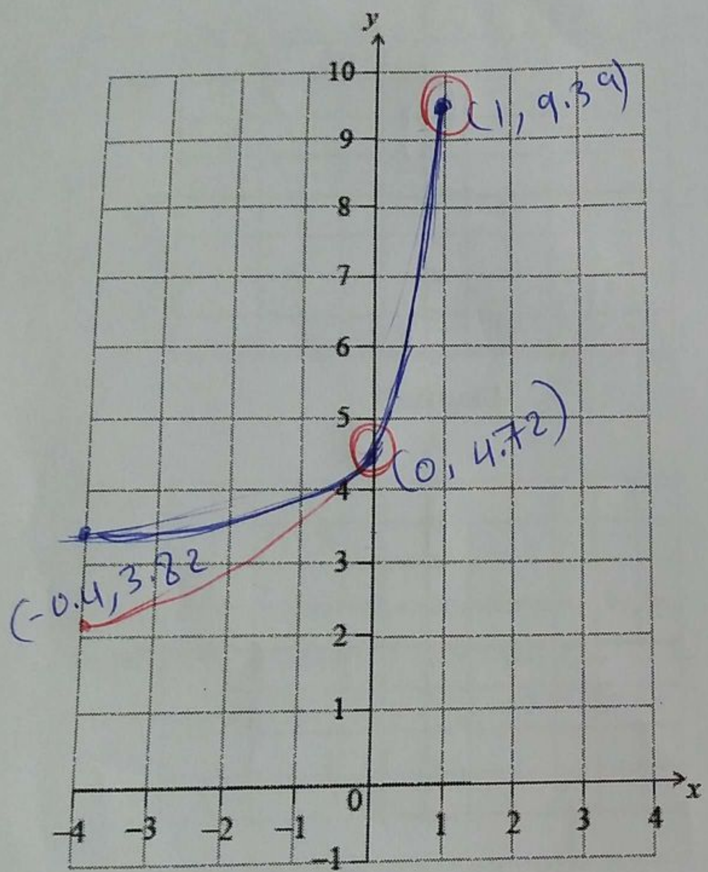
$g(x) = f(x+6) - 2$

6 units left

2 units down

8a. Let $f(x) = e^{x+1} + 2$, for $-4 \leq x \leq 1$.

On the following grid, sketch the graph of f. *[3 marks]*



8b. The graph of f is translated by the vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to obtain the graph of a function g.

Find an expression for $g(x)$. *[3 marks]*

$g(x) = e^{(x-3)+1} + 2 - 1$

$g(x) = e^{x-2} + 1$

9a. Consider the expansion of $(x+2)^{11}$.

[1 mark]

Write down the number of terms in this expansion.

n of terms = 11+1 = 12

[4 marks]

9b. Find the term containing x^2.

$\binom{11}{r} x^{11-r} (2)^r$ 1

$11-r=2$ 1

$r=9$

$\binom{11}{9} x^2 (2)^9$ 1

$55 \times 2^9 x^2 = 28160x^2$ 1

10a. Let $f(x)=2x+4$ and $g(x)=7x^2$.

[3 m

Find $f^{-1}(x)$.

$x = 2y+4$ 1

$\frac{x-4}{2} = y$ 1

$f^{-1}(x) = \frac{x-4}{2}$ 1

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10b. Find $(f \circ g)(x)$.

$(f \circ g)(x) = 2(7x^2) + 4$ 1

$= 14x^2 + 4$

1

[2 mar

10c. Find $(f \circ g)(3.5)$.

$f \circ g(3.5) = 14(3.5)^2 + 4$ 1

$= 175.5$

≈ 176 1 N2

11. Expand $\sum_{r=4}^{7} 2^r$ as the sum of four terms.

$2^4 + 2^5 + 2^6 + 2^7 = 240$

1

12a. Consider an infinite geometric sequence with $u_1 = 40$ and $r = \frac{1}{2}$.

[4 marks]

(i) Find u_4.

(ii) Find the sum of the infinite sequence.

(i)

$U_4 = U_1 r^{n-1}$ (1)

$= 40\left(\frac{1}{2}\right)^{4-1}$ (1)

$= \frac{40}{8} = 5$ N (2)

(ii)

$S_\infty = \frac{U_1}{1-r}$

$= \frac{40}{1-\frac{1}{2}}$ (1) $= 80$ (1) NV

12b. Consider an arithmetic sequence with n terms, with first term (-36) and eighth term (-8).

(i) Find the common difference.

[5 marks]

(ii) Show that $S_n = 2n^2 - 38n$.

1^{st} term $= -36$

8^{th} term $= -8$

$-8 = -36 + 7d$ (1)

~~-28~~ (1)

$28 = 7d$

$\boxed{d = 4}$ (1) (NV)

$S_n = \cancel{\frac{n}{2}(U_1 + U_n)}$ (1)

$= \frac{n}{2}(2U_1 + (n-1)d)$

$\underset{1}{-36} \overset{+d}{} \; \underset{2}{\_\_} \overset{+d}{} \; \underset{3}{\_\_} \overset{+d}{} \; \underset{4}{\_\_} \overset{+d}{} \; \underset{5}{\_\_} \overset{+d}{} \; \underset{6}{\_\_} \overset{+d}{} \; \underset{7}{\_\_} \overset{+d}{} \; \underset{8}{8}$

(2)

$S_n = \frac{n}{2}\left(2(-36) + (n-1)(4)\right)$

$= \frac{n}{\cancel{2}}\left(\cancel{-72}^{-36} + \cancel{4}^{2}n - \cancel{4}^{2}\right)$

$= 2n^2 - 38n$

12c. The sum of the infinite geometric sequence is equal to twice the sum of the arithmetic sequence.

[5 marks]

Find n.

$S_\infty = 80$

$S_n = \frac{1}{2} 80$ (1)

$= 40$

$S_n = \frac{n}{2}(2U_1 + (n-1)d)$

$40 = \frac{n}{2}(2(-36) + (n-1)4)$

$80 = n(-72 + 4n - 4)$

$40 = 2n^2 - 38n$ (1)

(1)

$20 = n^2 - 19n$

$(n-20)(n+1) = 0$

$n = 20$ (2) ← N3

$\cancel{n = -1}$

13a. In an arithmetic sequence $u_{10} = 8$, $u_{11} = 6.5$.

[1 mark]

Write down the value of the common difference.

$d = 6.5 - 8 = -1.5$ (1)

[3 marks]

13b. Find the first term.

$U_n = U_1 + (n-1)d$ (1)

$8 = U_1 + (10-1)(-1.5)$ (1)

$U_1 = 21.5$ (1)

[2 marks]

13c. Find the sum of the first 50 terms of the sequence.

$S_{50} = \frac{n}{2}(2U_1 + (n-1)d)$

$= \frac{50}{2}(2 \times 21.5 + (50-1)(-1.5))$ (1)

$= -762.5$ (1) N2

14a. The first three terms of a geometric sequence are $u_1 = 0.64,\ u_2 = 1.6$, and $u_3 = 4$.

Find the value of r.

[2 marks]

$$r = \frac{1.6}{0.64} \;1\; = 2.5 \;1$$

14b. Find the value of S_6.

[2 marks]

$$S_6 = \frac{0.64\left(2.5^{6}-1\right)}{2.5-1} \;1$$

$$= 103.74 \approx 104 \;1$$

14c. Find the least value of n such that $S_n > 75\,000$.

[3 marks]

$$\frac{0.64\left(2.5^{n}-1\right)}{2.5-1} > 75000 \;1$$

$$2.5^{n} = 128907$$

$$n = \log_{2.5} 128907 = \cancel{12.8478}\ 13.18 \;1$$

$$\boxed{n \approx \cancel{13}}\ 14 \;1$$

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