M02-03/17/MATHSL11/SP1/ENG/

22107305

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

IB MATH SL
GRADE 11
SEMESTER 2
VECTORS

Candidate name

Answer Key

THE UNIVERSAL AMERICAN SCHOOL
U·A·S
DUBAI

INSTRUCTION TO CANDIDATES

- Write your name in the box above.
- Write all your answers on the blank paper.
- Clearly identify each question by writing the the number/letter.
- Draw a horizontal line to separate questions.
- Make sure your work is clear and readable.
- Detach unused paper

IB Math SL - Vectors *[57 marks]*

Consider the points A (1 , 5 , 4) , B (3 , 1 , 2) and D (3 , k , 2) , with (AD) perpendicular to (AB) .

1a. Find [3 marks]

(i)

$\overrightarrow{AB}$;

(ii)

$\overrightarrow{AD}$ giving your answer in terms of k .

[3 marks]

B − A

i] $\overline{AB} = \begin{pmatrix} 3-1 \\ 1-5 \\ 2-4 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$

D − A

ii] $\overline{AD} = \begin{pmatrix} 3-1 \\ k-5 \\ 2-4 \end{pmatrix} = \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix}$

1b. Show that $k = 7$. → solve for K [3 marks]

AD ⊥ AB , So

$\overrightarrow{AD} \cdot \overrightarrow{AB} = \begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} = 4 + -4(k-5) + (-2)(-2) = 0$

$8 - 4(k-5) = 0$

$8 = 4(k-5)$

$2 = k-5$

$\boxed{7 = k}$

1c. The point C is such that

$\overrightarrow{BC} = \frac{1}{2}\overrightarrow{AD}$.

Find the position vector of C.

$C - B$

$$\begin{pmatrix} x_c - 3 \\ y_c - 1 \\ z_c - 2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$$

$x - 3 = \frac{1}{2} \times 2$

$\boxed{x = 4}$

$y_c - 1 = \frac{1}{2} \times 2$

$\boxed{y_c = 2}$

$z_c - 2 = \frac{1}{2}(-2)$

$\boxed{z_c = +1}$

$$\overrightarrow{OC} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$$

point $C = (4, 2, 1)$

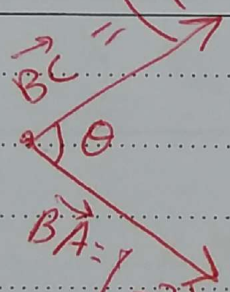
Orgin $= (0, 0, 0)$

$$\overrightarrow{OC} \begin{pmatrix} 4-0 \\ 2-0 \\ 1-0 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$$

position vector

1d. Find $\cos A\hat{B}C$. [3 marks]

$$\overrightarrow{BC} = \begin{pmatrix} 4-3 \\ 2-1 \\ 1-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$



$$\overrightarrow{BA} = \begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix}$$

$$\cos A\hat{B}C = \frac{\overrightarrow{BC} \cdot \overrightarrow{BA}}{|\overrightarrow{BC}|\,|BA|} = \frac{-2+4-2}{\sqrt{3}\ \sqrt{24}}$$

$$= \frac{0}{\sqrt{3}\sqrt{24}} = 0$$

$\therefore \cos A\hat{B}C = 0$

Didn't ask for the angle value

only $\cos A\hat{B}C$

2. The following diagram shows quadrilateral ABCD, with

$\overrightarrow{AD} = \overrightarrow{BC}$,

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, and

$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$.

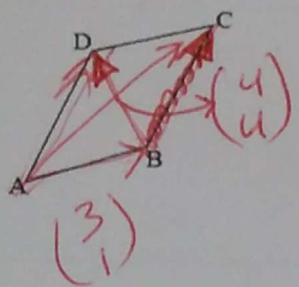


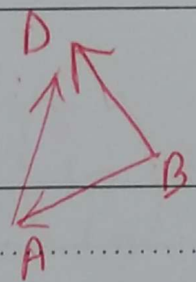
diagram not to scale

2a. Find $\overrightarrow{BC}$. [2 marks]

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= -\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

2b. Show that $\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$. [2 marks]



$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD}$

$= -\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

2c. Show that vectors $\overrightarrow{BD}$ and $\overrightarrow{AC}$ are perpendicular.

$\overrightarrow{BD} \cdot \overrightarrow{AC} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 4 \end{pmatrix} = -8 + 8 = 0$

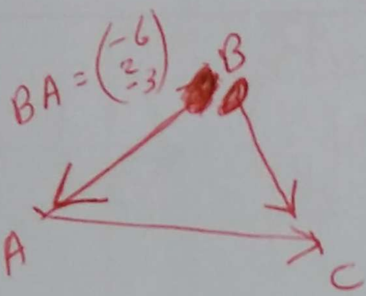
$\therefore \cos\theta = 0$

$\therefore \theta = 90$

3. Let

$\overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and

$\overrightarrow{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.



3a. Find $\overrightarrow{BC}$. [2 marks]

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -8 \\ -1 \\ -1 \end{pmatrix}$

3b. Find a unit vector in the direction of $\overrightarrow{AB}$. [3 marks]

$\overrightarrow{AB}_{unit} = \frac{1}{|\overrightarrow{AB}|} \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$

$= \frac{1}{7} \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6/7 \\ -2/7 \\ 3/7 \end{pmatrix}$

$|AB| = \sqrt{(6)^2 + (-2)^2 + 3^2}$

$= \sqrt{36 + 4 + 9}$

$= \sqrt{49} = 7$

3c. Show that

$\overrightarrow{AB}$ is perpendicular to

$\overrightarrow{AC}$.

[3 marks]

$\overrightarrow{AB} \perp \overrightarrow{AC}$

$\overrightarrow{AB} \cdot \overrightarrow{AC} =$

$\begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix} = -12 + 6 + 6 = 0$

4a. Let $\boldsymbol{u}$

[3 marks]

$\vec{u} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and [redacted]

$\vec{w} = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix}$. Given that $\boldsymbol{u}$ is perpendicular to $\boldsymbol{w}$, find the value of p .

$\vec{u} \cdot W = 0 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix} = 6 - 3 - P = 0$

$\boxed{3 = P}$

4b. Let

[3 marks]

$v = \begin{pmatrix} 1 \\ q \\ 5 \end{pmatrix}$. Given that

$|v| = \sqrt{42}$, find the possible values of

q .

$|V| = \sqrt{1^2 + q^2 + 5^2} = \sqrt{42}$

$\sqrt{26 + q^2} = \sqrt{42}$

$26 + q^2 = \pm 42$

26

$\sqrt{q^2} = \sqrt{\pm 16}$

$\boxed{q = \pm 4}$

Important

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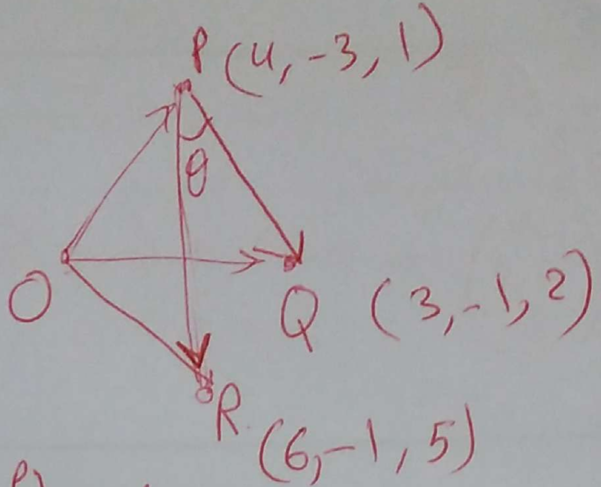
5. The vertices of the triangle PQR are defined by the position vectors

$$\overrightarrow{OP} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix},$$

$$\overrightarrow{OQ} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \text{ and}$$

$$\overrightarrow{OR} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}.$$



5a. Find [3 marks]

(i) $\overrightarrow{PQ}$;

(ii) $\overrightarrow{PR}$.

i) $\overrightarrow{PQ} = \underset{Q-P}{\begin{pmatrix} 3-4 \\ -1--3 \\ 2-1 \end{pmatrix}} = \begin{pmatrix} -1 \\ +2 \\ 1 \end{pmatrix}$

ii) $\overrightarrow{PR} = \underset{R-P}{\begin{pmatrix} 6-4 \\ -1--3 \\ 5-1 \end{pmatrix}} = \begin{pmatrix} 2 \\ +2 \\ 4 \end{pmatrix}$

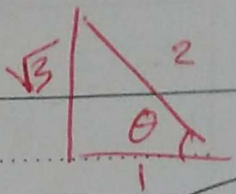
5b. Show that [7 marks]
$\cos R\hat{P}Q = \frac{1}{2}$.

$$\cos R\hat{P}Q = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PQ}|\,|\overrightarrow{PR}|} = \frac{-2+4+4}{\sqrt{6}\ \sqrt{24}} = \frac{6}{\sqrt{24\times 6}}$$

$$= \frac{6}{\sqrt{24\times 6}} = \frac{6}{\sqrt{6\times 4\times 6}} = \frac{6}{2\sqrt{6^2}} = \frac{1}{2}$$

5c. (i) Find $\sin R\hat{P}Q$.

(ii) Hence, find the area of triangle PQR, giving your answer in the form $a\sqrt{3}$.



i) $\cos R\hat{P}Q = \frac{1}{2}$

$\therefore \sin R\hat{P}Q = \frac{\sqrt{3}}{2}$

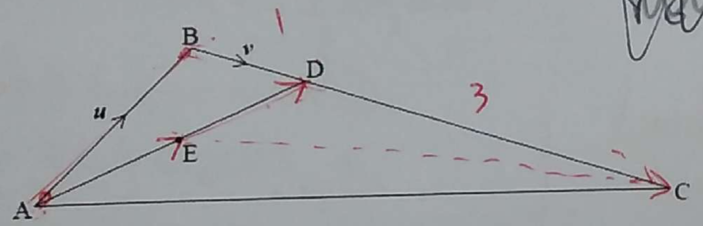
ii) $A_{\triangle} = \frac{1}{2}|\vec{PR}||\vec{PQ}|\sin\theta$

$= \frac{1}{2}\sqrt{6}\cdot\sqrt{24}\,\frac{\sqrt{3}}{2}$

$= \frac{1}{2}(12)\frac{\sqrt{3}}{2}$

$= 3\sqrt{3}$

6. In the following diagram, $u = \overrightarrow{AB}$ and $v = \overrightarrow{BD}$.



The midpoint of $\overrightarrow{AD}$ is E and $\frac{BD}{DC} = \frac{1}{3}$.

Express each of the following vectors in terms of u and v.

[3 marks]

6a. $\overrightarrow{AE}$

$\overrightarrow{AE} = \frac{1}{2}\overrightarrow{AD} = \frac{1}{2}(u + v)$

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

6b) $\overrightarrow{EC} = \overrightarrow{ED} + \overrightarrow{DC}$

$= \frac{1}{2}(\vec{u} + \vec{v}) + 3\vec{v}$

$= \frac{1}{2}\vec{u} + \frac{1}{2}\vec{v} + 3\vec{v}$

$= \frac{1}{2}\vec{u} + 3.5\vec{v}$

$\overrightarrow{ED} = \overrightarrow{AE}$

$\overrightarrow{DC} = 3\overrightarrow{BD}$

Important