

Answer Key



10

Test Date: Monday June 12th, 8:30 – 10:00

Class: Math Grade 10

EXAM

Boundaries

Total Marks	1	2	3	4	5	6	7
86	0	18	26	39	48	61	74

Congratulations on a great year.

It has been an honor to have you as students and I wish you all the best in preparing for your Exam.

The following is a comprehensive list of topics you should review, as well as many practice problems, and even some copies of your old tests.

You should use this in combination with your class notes and old quizzes and tests.

Functions

$g(x) = \frac{2x+3}{x-2}$	$f(x) = 2x^2 + x - 7$	$h(x) = \frac{12}{x}$	$j(x) = 2x + 9$
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a) What is $g(10)$ $\frac{2(10)+3}{10-2} = 23/8$

b) What is $f(-3)$ $2(-3)^2 + (-3) - 7 = 8$

c) What is $h(\frac{-3}{2})$ $\frac{12}{-3/2} = -8$

d) What is $j^{-1}(7)$
 $\Rightarrow j(x) = 2x + 9 \parallel \frac{x-9}{2} = y$
 $\xrightarrow{j^{-1}(7)} x = 2y + 9 \parallel y = \frac{7-9}{2} = -1$

e) What is $f(h(4))$
 $h(4) = \frac{12}{4} = 3$ $f(3) = 14$
 $f(3) = 2(3)^2 + (3) - 7$

Systems of Equations (Simultaneous Equations) – Practice Questions

Solve the following systems of equations

$4x + y = 2$ and $x - y = 3$

$x = 1$ $y = -2$

$y = x^2$ and $y = 8 - x^2$

$x = -2 \mid y = 4$
 $x = 2 \mid y = 4$

$y = -x - 3$ and $x^2 + y^2 = 17$

$x = -4 \mid y = 1$

$y = \frac{1}{2}x - 5$ and $y = x^2 + 2x - 15$

$x = 1 \mid y = -4$

$x = -4 \mid y = -7$

$x = 2.5 \mid y = -3.75$ ₃

Systems of Equations (Simultaneous Equations) – Worded Question

There are 30 balls in a bowl red or blue.

Let x = number of red balls

Let y = number of blue balls

Red balls weigh 15g. Blue balls weigh 23g.

- 1.) Write an equation, in terms of x and y , for the number of balls in the bowl.

$$x + y = 30$$

The total weight of balls is 626g.

- 2.) Write an equation, in terms of x and y , for the total weight of all the balls in

$$15x + 23y = 626$$

- 3.) Write down this system of two equations.

$$\begin{aligned} x + y &= 30 \\ 15x + 23y &= 626 \end{aligned}$$

- 4.) Solve to find the number of red and blue balls.

$$x = 8$$

$$y = 22$$

Asymptotes – Practice Question

$$f(x) = \frac{6}{x+5} - 4$$

1.) Does this function have asymptotes? Why?

Yes, Rational function

2.) How many asymptotes does this function have? Why?

2, Rational function

3.) What are the equations of the asymptotes of this function?

VA: $x = -5$

HA: $y = -4$

4.) What are the co-ordinates of the intercepts with axes? Show your calculations.

x_{int}

$y = 0$

$$\frac{6}{x+5} - 4 = 0$$

$x+5$

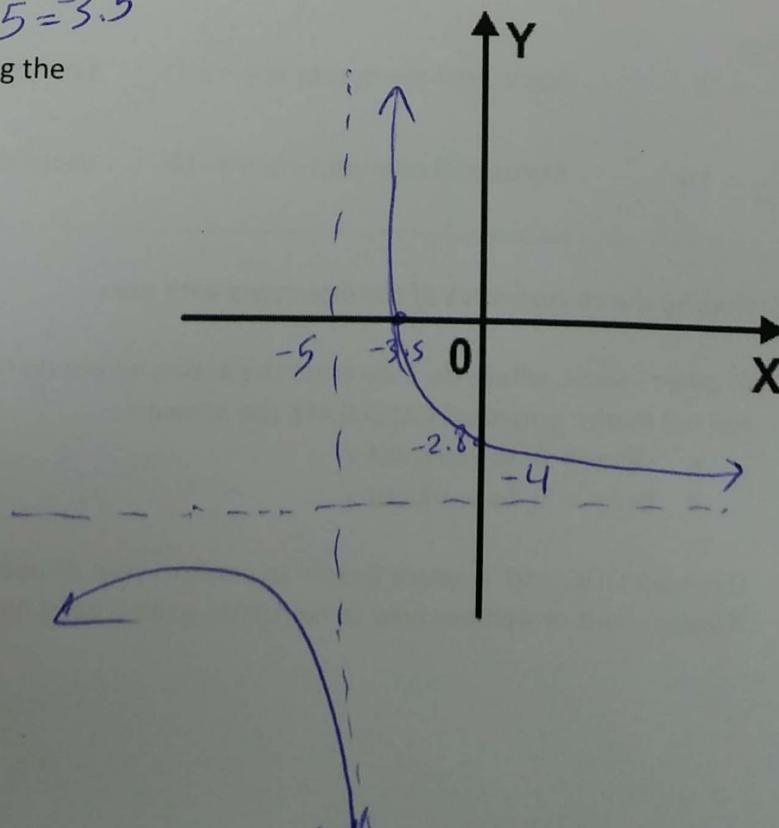
$$x = \frac{6}{4} - 5 = -3.5$$

y_{int}

$x = 0$

$$y = \frac{6}{0+5} - 4 = -2.8$$

5.) Sketch the graph, using the information above



Quadratics Equations – Quadratic Formula

Quadratic Formula

This will be given to you

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Practice Questions

Solve the following equations. Give EXACT answers.

(In other words, DO NOT use decimals. Fractions and radicals (\sqrt{x}) only)

1.) $(3x - 1)(x + 5) = 0$

$$x = \frac{1}{3}, x = -5$$

2.) $2x^2 - 2x - 5 = 0$

$$x = \frac{1 \pm \sqrt{11}}{2}$$

Completing the Square

$$\left(x + \frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$

!! WARNING !! This will NOT be given to you !!

Practice questions

Solve by completing the square

1.) $2x^2 - 8x + 18 = 0$

No solution

2.) $5x^2 + 15x - 30 = 0$

$$x = \frac{-3 \pm \sqrt{33}}{2}$$

$$\begin{aligned} 5x^2 + 15x - 30 &= 0 \\ x^2 + 3x - 6 &= 0 \\ \left(x + \frac{3}{2}\right)^2 &= -6 + \left(\frac{3}{2}\right)^2 \end{aligned}$$

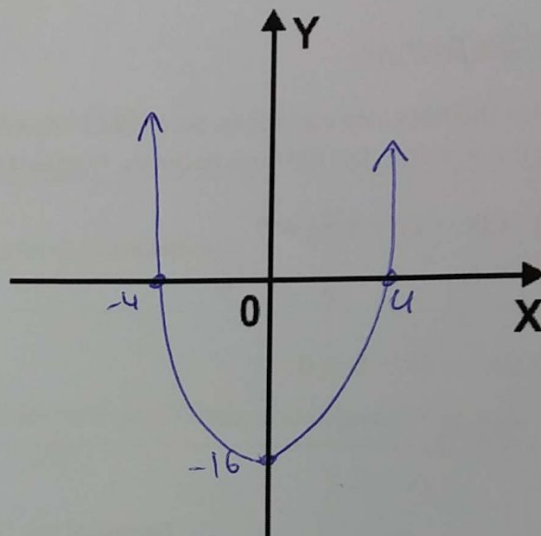
$$x = \pm \sqrt{\frac{33}{4}} - \frac{3}{2} = \frac{-3 \pm \sqrt{33}}{2}$$

Sketching Graphs

- Marks are awarded for:
 - The shape of the graph
 - The x intercepts, and y intercepts
 - Asymptotes (if there are any)

$$y = x^2 - 16$$

Clearly show the locations of all intercepts



$$y = 3 + \frac{2}{x+1}$$

Clearly show the locations of the asymptotes and the y-intercept)

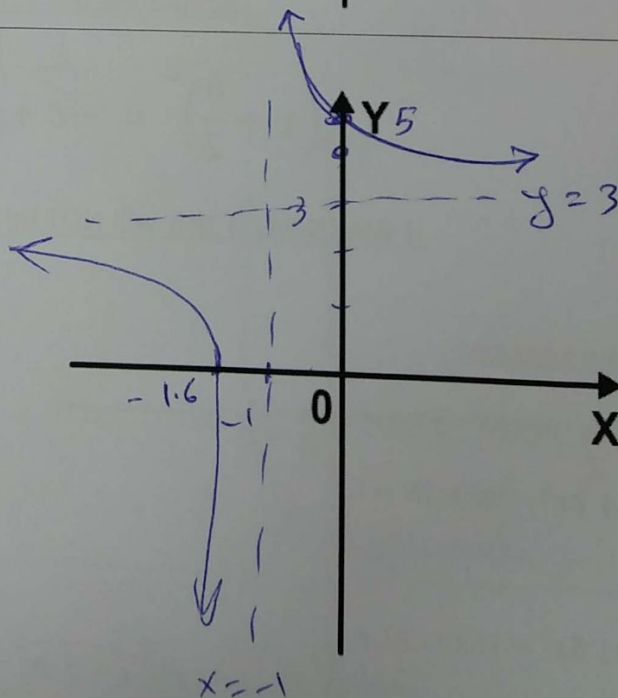
$$HA: y=3$$

$$VA: x=-1$$

x_{int}

$$x=0$$

$$y = 3 + \frac{2}{0+1} = 5$$



Index Laws

- $a^m \times a^n = a^{m+n}$

- $\frac{a^m}{a^n} = a^{m-n}$

- $(a^m)^n = a^{m \times n}$

- $a^{-n} = \frac{1}{a^n}$

and

- $\frac{1}{a^{-n}} = a^n$

Practice Questions

Simplify the following, leave your solution WITHOUT negative exponents.

(Hint: How do you get rid of negative exponents?)

$$\frac{x^2 y^{-2} z^2}{x^2 y^3 z^{-1}} = \frac{z^3}{y^5}$$

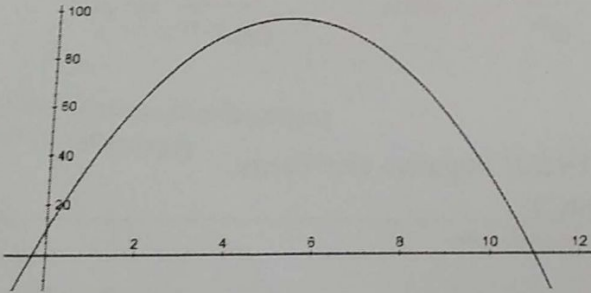
$$\frac{x^2 y^3 z^6}{x^4 y^6 z^1} = \frac{z^5}{x^2 y^3}$$

$$\frac{x^{\frac{1}{2}} x^{\frac{1}{3}} z^{-1}}{y^2 y^3 z^2} = \frac{x^{\frac{5}{6}}}{y^5 z^3}$$

Quadratic Equations – Worded Question

A projectile is thrown up in the air. The height, h meters, of the projectile above the ground after t seconds, is given by:

$$h = -3t^2 + 32t + 10$$



The graph will NOT be given to you.
Use your GDC to get this.

- 1.) What is the initial height? $h_i = 10$
- (What is this question asking?)
 - (Hint: NO calculations are required to answer this! You don't even need the graph)

- 2.) How many seconds is the projectile above 60 meters?

- (What information do you need to answer this?)

after 1.90 and $t = 8.77$

$$60 = -3t^2 + 32t + 10$$
$$t = 8.77 \quad t = 1.9$$

- 3.) What is the highest height the projectile will go? Show your workings.

- For full marks, you need CALCULATE the answers.
- Use your GDC, and 'Analyze Graph' to confirm your answer.
- If you cannot remember how to calculate, write 'I used my GDC to get the answer.'

Vertex =

$$x = \frac{-b}{2a} = \frac{-32}{2(-3)} = 5.33 = \frac{32}{6}$$

$$y = -3\left(\frac{32}{6}\right)^2 + 32\left(\frac{32}{6}\right) + 10$$
$$= 95.3 \text{ m}$$

Sequences and Series

The n th term of an arithmetic sequence

$$u_n = u_1 + (n-1)d$$

The sum of n terms of an arithmetic sequence

$$S_n = \frac{n}{2}[2u_1 + (n-1)d] = \frac{n}{2}(u_1 + u_n)$$

The n th term of a geometric sequence

$$u_n = u_1 r^{n-1}$$

Question 1 (2 Marks)

a) i) Is the function $f(x) = 3x - 5$ a one-to-one or a many-to-one function? (1)

ii) Explain why you chose the answer above.

one to one.
wt every x value there is only 1 y value.

Question 2 (6 Marks)

$$f(x) = 3x^2 + x \text{ and } g(x) = x - 3$$

a) Explain, in words, the difference between $f(x) + g(x)$ and $(f \times g)(x)$. (2)

adding function multiplying

b) Determine $(f \times g)(2)$, in simplest form. (2)

$3(x^2) + x$ $f(2) = 3(2^2) + 2 = 14$ $(f \times g)(2) = 14(-1) = -14$

c) Determine the value of $f(g(3))$. (2)

$g(3) = 3 - 3 = 0$

$f(0) = 3(0)^2 + 0 = 0$

Question 3 (10 Marks)

Consider the function $f(x) = \frac{2}{x+2} + 2$.

- a) i) Write down the equations of the asymptotes of this function.

VA: $x = -2$

HA: $y = 2$

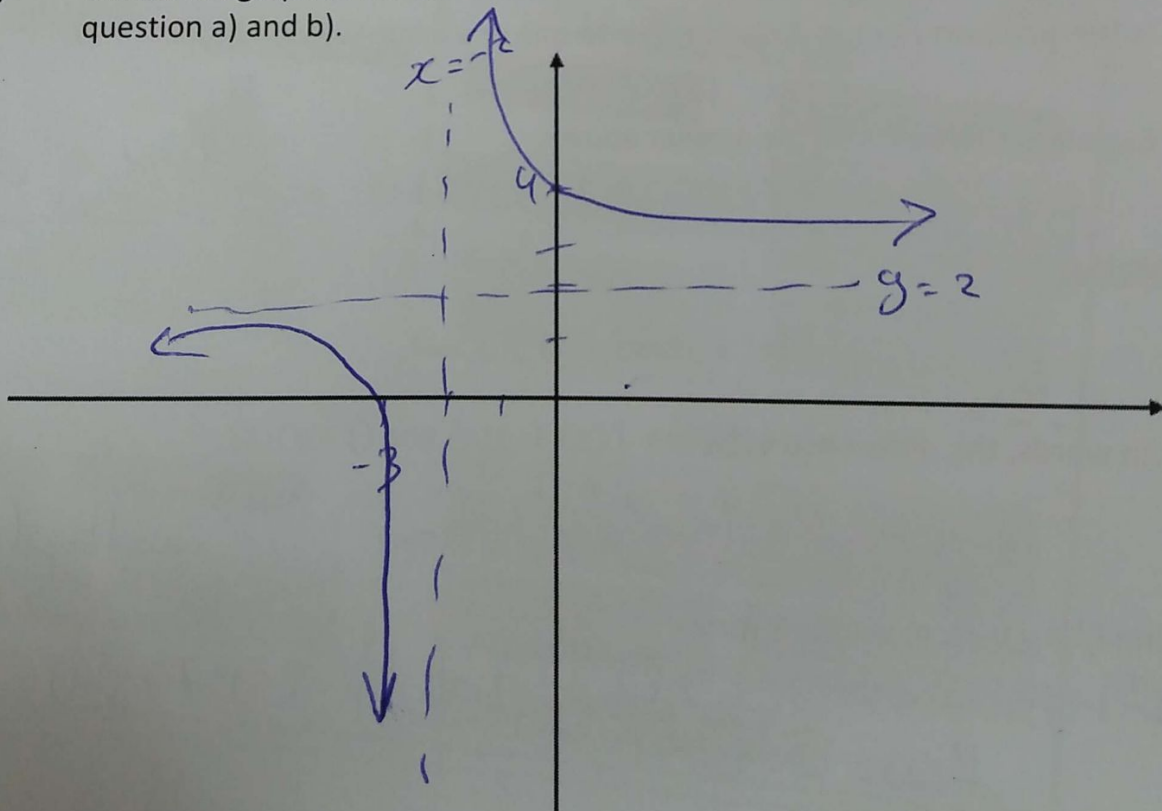
- ii) Give a brief explanation why this function has asymptotes.

Rational function

- b) Determine the coordinates of the intercepts with axes.

$$0 = \frac{2}{x+2} + 2 \Rightarrow x = \frac{2}{-2} - 2 = -3 \quad \left| \quad \begin{array}{l} \text{y int} \\ y = \frac{2}{0+2} + 2 \\ = 1 + 3 = 4 \end{array} \right.$$

- c) Sketch the graph of the function above and indicate the features you calculated question a) and b).



Question 4 (10 Marks)

Consider the function $f(x) = 3^x - 2$.

- a) i) Explain why this function will have an asymptote. (2)
exponential
- ii) Will this asymptote be horizontal or vertical? (1)
Horizontal
- b) i) At which value will this graph pass through the y-axis? (1)
 $y = 3^0 - 2 = 1 - 2 = -1$
- ii) Estimate a value where this graph will pass through the x-axis. (1)
 $0 = 3^x - 2 \Rightarrow 3^x = 2 \Rightarrow \log_3 2 = x$
 $x = 0.631$
- c) Determine the value of $f(-1)$, giving your answer as a fraction. (2)
 $3^{(-1)} - 2 = \frac{1}{3} - 2 = \frac{-5}{3}$
- d) Determine the value of x for which $f(x) = 25$. Give some calculation/explanation of how you get to the answer. (3)
 $25 = 3^x - 2$
 $3^x = 23 \Rightarrow \log_3 23 = 2.85$

Question 5 (8 Marks)

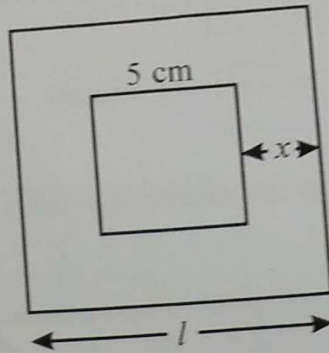
A group of leopards is introduced into a game park in Tanzania. After t years, the number of leopards, N is given by:

$$N = 10 \cdot (2)^t$$

- a) How many leopards were introduced to the game park initially? (1)
 $N_0 = 10(2^0) = 10$
- b) How many leopards will there be in the park after 2 years? (2)
 $N_2 = 10(2^2) = 40$
- c) How long will it take the leopard population to reach 40? (3)
Give some calculation/explanation of how you get to the answer.
 $40 = 10(2^t) \Rightarrow 4 = 2^t$
 $2^2 = 2^t \Rightarrow t = 2 \text{ yrs}$
- d) Will this be an accurate model of the leopard population over a very long time? (2)
Give a reason for your answer.
*No, some will die
some will not have offsprings*

Question 6 (6 Marks)

A picture is in the shape of a square of side 5 cm. It is surrounded by a wooden frame of width x cm, as shown in the diagram below.



The length of the wooden frame is l cm, and the area of the wooden frame is A cm².

- (a) Write an expression for the length l in terms of x .

$$l = 2x + 5$$

- (b) Write an expression for the area A in terms of x .

$$A = l^2 - 5^2 = (2x + 5)^2 - 5^2$$

- (c) If the area of the frame is 24 cm², find the value of x .

$$24 = (2x + 5)^2 - 25$$

$$49 = (2x + 5)^2$$

$$\pm \sqrt{49} = 2x + 5$$

$$\pm \frac{\sqrt{49} - 5}{2} = x$$

$$x = 1$$

Question 7 (4 Marks)

The table below shows graphs of 9 different functions.

1.	2.	3.
4.	5.	6.
7.	8.	9.

Match a graph from the table above with each of the descriptions given below. Write the number of the graph after each description.

a) The graph of $f(x) = 3 \cdot \left(\frac{1}{3}\right)^x$ (2)
 9

b) The graph $y = \frac{4}{x-1} - 2$ (2)
 1

Question 8 (4 Marks)

Let $f(x) = \frac{2}{(x-3)(x+2)} + 3$

a) Is $f(x)$ one-to-one or many-to-one?

(1)

b) List all asymptotes of $f(x)$ and state if they are horizontal or vertical.

(3)

VA = $x=3$, $x=-2$

HA = $y=-3$

(3)

c) Calculate the x and y intercepts of $f(x)$ if they exist.

$\frac{2}{-3} = (x^2 - x - 6)$

x int $y=0$

y int $x=0$

$x=2.86$

Question 1 (9 Marks)

Consider the following function:

$f(x) = 3\left(\frac{2}{3}\right)^x - 2, x \in [-1, 4]$

$y = \frac{2}{(-3)(2)} + 3$ $y = 2.67$

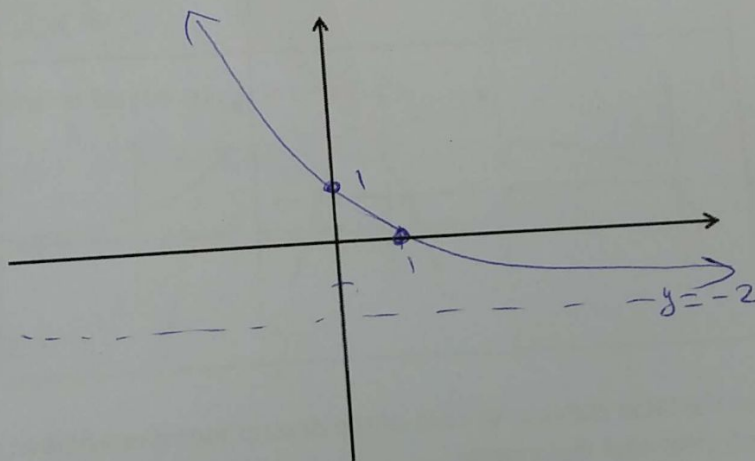
a) Draw the graph of $f(x)$ for the given domain, indicating the values of the intercepts with the axes, as well as the asymptote(s).

(4)

HA = $y=-2$

Y int: $x=0$
 $y = 3\left(\frac{2}{3}\right)^0 - 2$
 $= 3 - 2 = 1$

X int $y=0$
 $3\left(\frac{2}{3}\right)^x = 2$
 $x=1$



b) Write down the range of $f(x)$.

(2)

c) Determine the value of $f(2)$.

(1)

d) Determine the value of x if $f(x) = -1$ (correct to 2 decimal places).

(2)

$-1 = 3\left(\frac{2}{3}\right)^x - 2$

$\log_{2/3} \frac{1}{3} = x$

$\frac{1}{3} = \left(\frac{2}{3}\right)^x$

$x = 2.71$

Question 2 (6 Marks)

Let $f(x) = \left(\frac{2}{3}\right)^x$

Describe the following transformations

a) $f(x) = \left(\frac{2}{3}\right)^x + 2$ up 2 units

(1)

b) $f(x) = \left(\frac{2}{3}\right)^{(x-1)}$ right 1 unit

(1)

c) $f(x) = -\left(\frac{2}{3}\right)^x$ reflect over x-axis

(1)

d) $f(x) = -\left(\left(\frac{2}{3}\right)^{x-1} + 2\right)$

up 2 units
right 1 unit
reflect over x-axis.

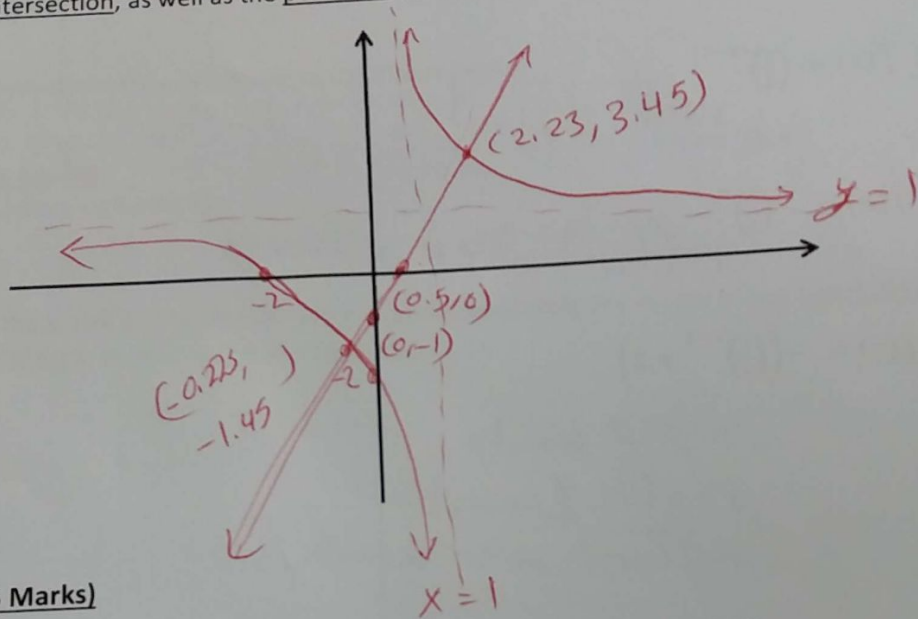
(3)

Question 3 (6 Marks)

Consider the following two functions:

$$f(x) = \frac{x+2}{x-1} \text{ and } g(x) = 2x - 1$$

On the axes given below, sketch the graphs of $f(x)$ and $g(x)$, showing the coordinates of the point(s) of intersection, as well as the position of asymptotes.



Question 4 (5 Marks)

A pole-vaulter runs up, attempting to clear a bar which is at a height of 5.80m.

He plants the pole a certain distance away from the mat hoping to reach his maximum height when he reaches the bar.

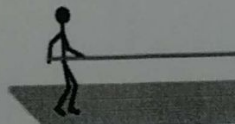
His height, at t seconds after the plant, is given by $h = -2.908t^2 + 8.143t$

- a) Would his maximum height be high enough to clear the bar? Provide an explanation/calculation to support your choice. (3)

$$\text{Vertex} = t = \frac{-b}{2a} = \frac{-8.143}{2(-2.908)} = 1.40$$

$$h(1.4) = -2.908(1.4)^2 + 8.143(1.4) = 5.70$$

- b) How long after his take-off would he land on the mat? (2)

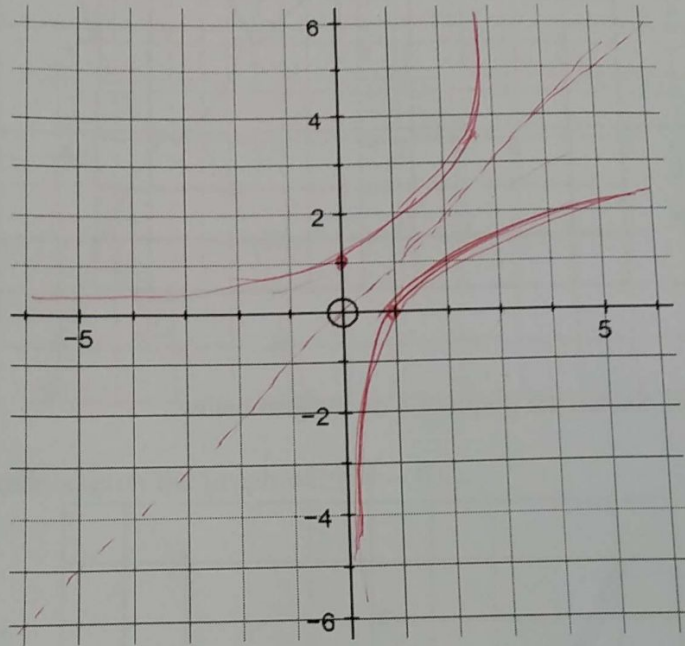


Question 6 (3 Marks)

$w = 2.7^{0.5t}$ is an exponential function that shows the weight of a bacterium, in μg , for the time $0 \leq t \leq 3.5$ hours.

On the grid provided below, graph the inverse of this function, taking into consideration the given domain.

(3)



Question 7 (6 Marks)

Let $f(x) = 7 - 2x$ and $g(x) = x + 3$.

(a) Find $g(f(x))$. ~~$7 - 2x$~~ $(7 - 2x) + 3$ (2)

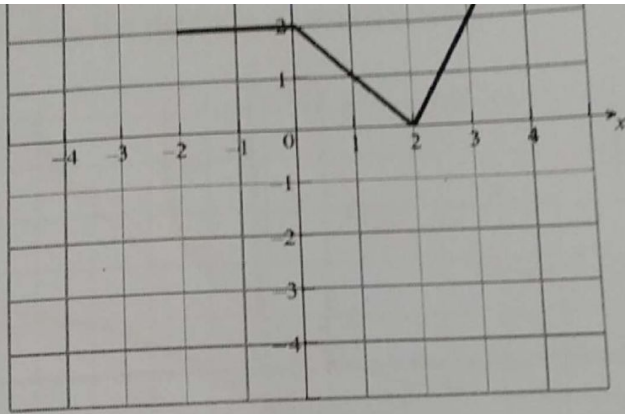
$= 10 - 2x$

(b) Write down $g^{-1}(x)$. $= x - 3$ (2)

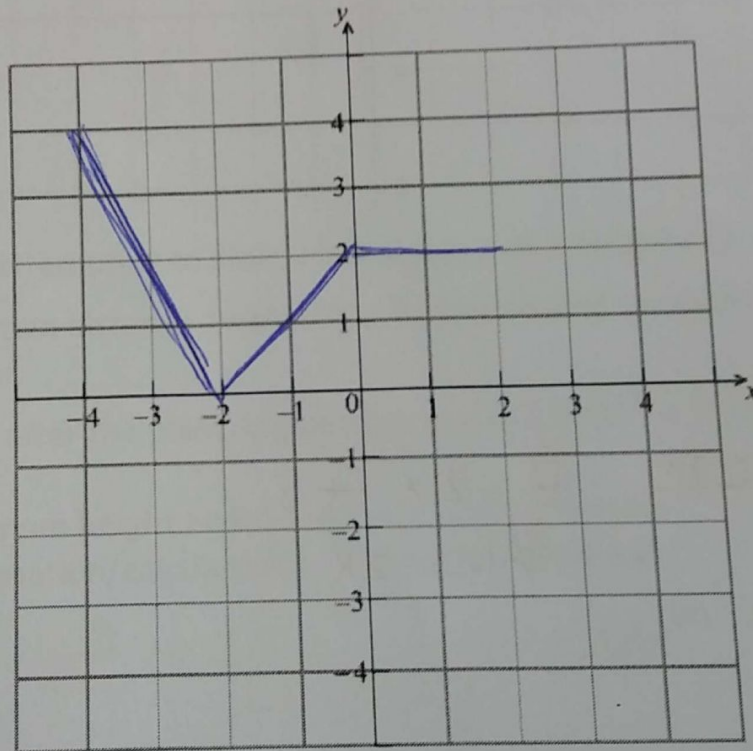
(c) Find $f(g^{-1}(5))$. (2)

$g^{-1}(5) = 5 - 3 = 2$

$f(2) = 7 - 2(2) = 3$



- (a) Let $h(x) = f(-x)$. Sketch the graph of h on the grid below.



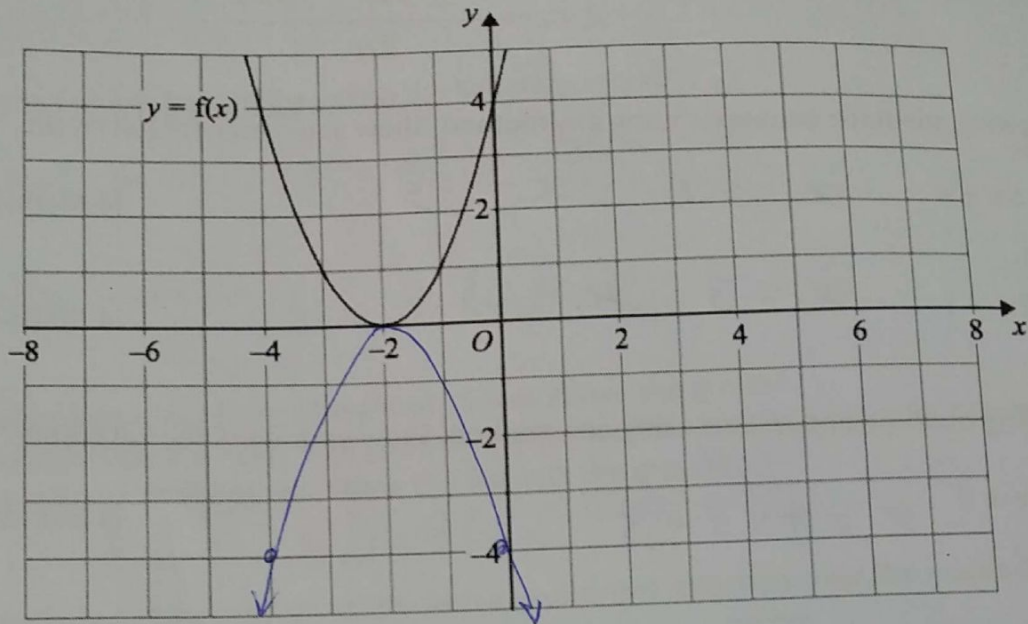
- (b) Let $g(x) = \frac{1}{2}f(x-1)$. The point $A(3, 2)$ on the graph of f is transformed to the point P on the graph of g . Find the coordinates of P . (3)

$$P = (4, 1)$$

Question 9 (3 Marks)

$y = f(x)$

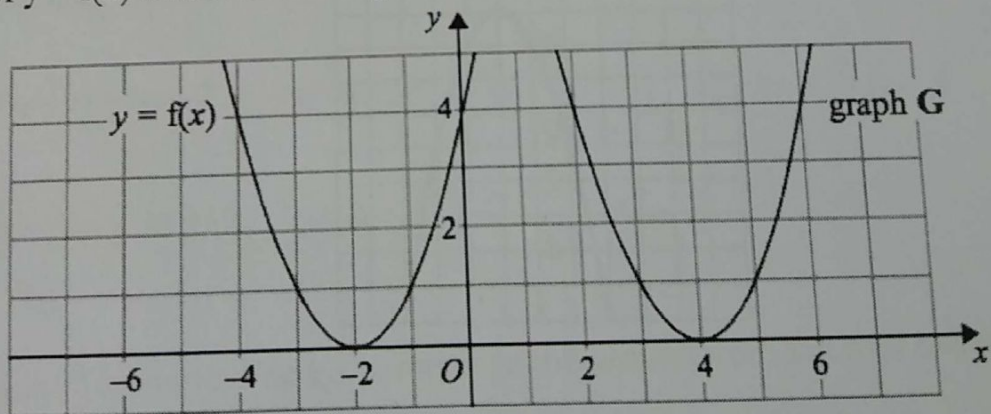
The graph of $y = f(x)$ is shown on the grid.



(a) On the grid above, sketch the graph of $y = -f(x)$.

(2)

The graph of $y = f(x)$ is shown on the grid.



The graph G is a translation of the graph of $y = f(x)$.

(b) Write down the equation of graph G.

(1)

$G(x) = f(x - 6)$

Completing the square

$$\left(x + \frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve the following quadratic equations using any method. Show your work for full credit.

1.) $0 = x^2 + 5x + 6$ $x = -2, x = -3$ (4 Marks)

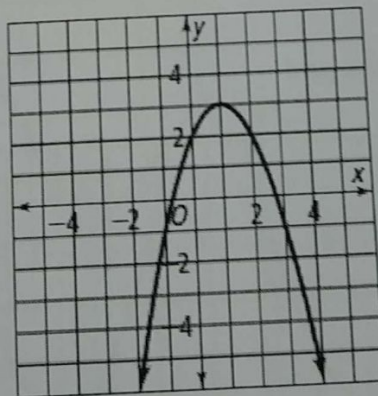
2.) $2 = x^2 - 2x - 1$ $x = 3, x = -1$ (4 Marks)

Solve the following quadratic equations using any method. Show your work for full credit.

3.) $0 = x^2 + 6x + 2$ $x = \pm\sqrt{7} - 3$ (4 Marks)

4.) $x^2 = 7$ $x = \pm\sqrt{7}$ (4 Marks)

5.) What is the vertex of the graph? Is it a minimum or a maximum? (3 Marks)



- 6.) What is the order, from widest to narrowest graph, of the following quadratic functions: (2 Marks)

$f(x) = -0.5x^2$
widest

$f(x) = \frac{1}{4}x^2$

$f(x) = x^2$
narrowest

Solve the following using Complete the Square method

7.) $0 = x^2 + 6x + 8$ $x = -2, x = -4$ (4 Marks)

8.) $0 = 2x^2 - 4x + 6$ No solution (4 Marks)

- 9.) Kyle drops a ball from a height of 15 feet above the ground. The function $h = -16t^2 + 15$ gives the height h of the ball (in feet) after t seconds. Determine at about what time the ball hits the ground. (4 Marks)

$t = \frac{1}{16} \sqrt{15} \Rightarrow 0.968s$ $h(t) = 0$

- 10.) Jared is casting his fishing line with a lead sinker attached over the edges of a pier. The pier is 11 feet above the water. The function $h = -12t^2 + 11$ gives the sinker's height h above the water (in feet) after t seconds. **Graph the function, including vertex and intercepts.** How many seconds does it take for the sinker to hit the water? (6 Marks)

$t = \sqrt{\frac{11}{12}} = 0.957$

Solve using any method

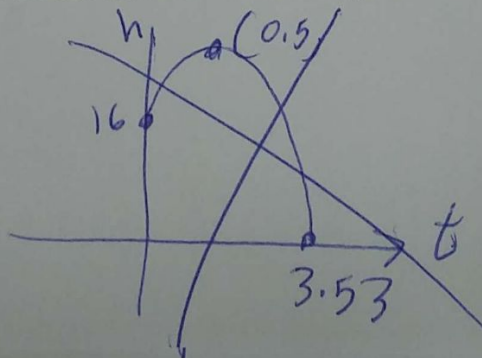
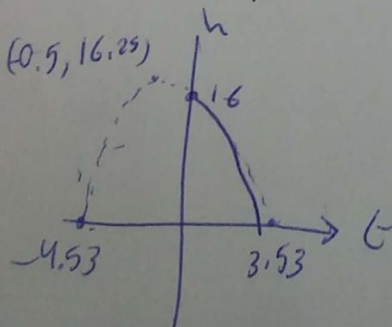
11.) $f(x) = \frac{3}{2}x^2 + 6x + 2$ $x = -0.367, x = -3.63$

12.) $f(x) = \frac{1}{4}x^2 + 4x - 10$ $x = 2.20, x = -18.2$

(4 marks each)

- 13.) A roofer is going to drop his hammer to the ground from the roof (after making sure the area is clear!) The roof is 16 feet high. The function $h = -t^2 - t + 16$ gives the hammer's height h above the ground (in feet) after t seconds.

Graph the function, including vertex and intercepts. How many seconds does it take for the hammer to hit the ground? (6 Marks)



Question 1: Simplify the following, write solutions without negative exponents.

a) $\frac{x^2 y^2 z^3}{x^{\frac{1}{2}} y^1 z^{\frac{2}{3}}}$ = $\frac{x^{1.5} y z^{\frac{7}{3}}}{}$ (3)

b) $(x^{\frac{1}{2}})^{\frac{3}{2}}$ $x^{\frac{3}{4}}$ (2)

c) $(x^2 y^2 z^{\frac{1}{3}})^2 (x^3 y^{\frac{1}{2}}) = x^5 y^{2.5} z^{\frac{1}{3}}$ (3)

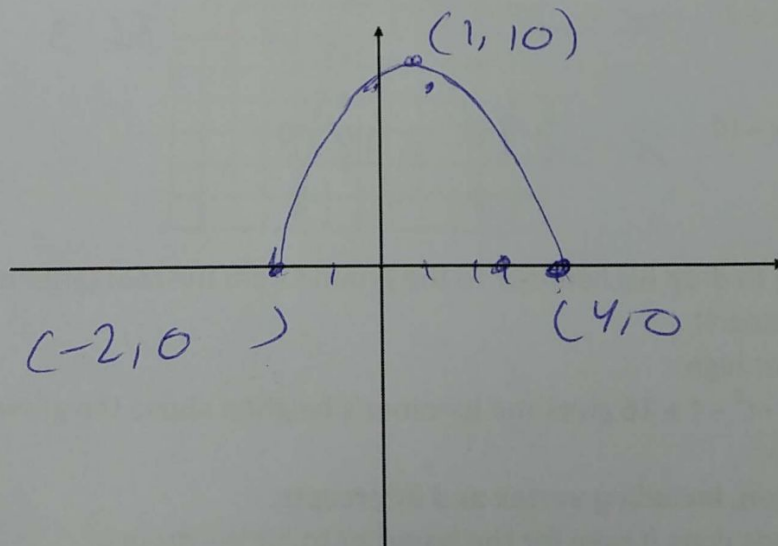
Question 2:

A quadratic function $f(x) = ax^2 + bx + c$ has the following features:

$a < 0$ and $c > 0$ and contains the following points

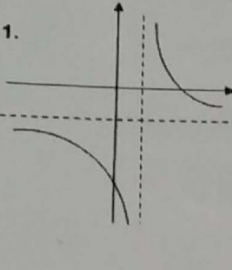
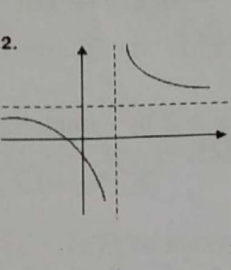
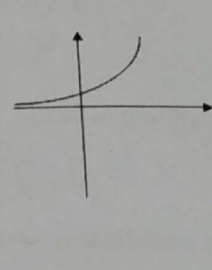
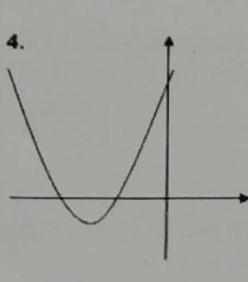
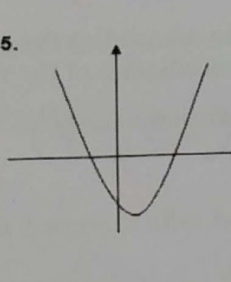
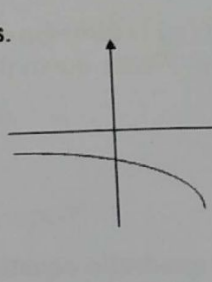
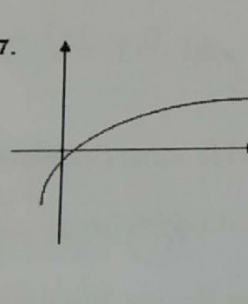
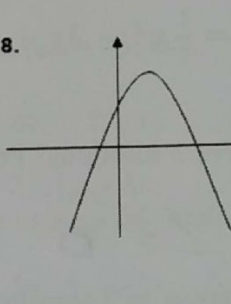
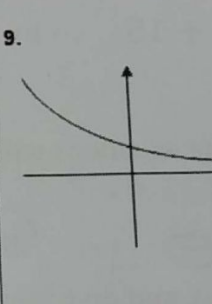
$(-2, 0)$, $(4, 0)$, $(1, 10)$

Sketch the graph of $f(x)$, and only include the features mentioned above. (4)



Question 3:

The table below shows graphs of 9 different functions.

1. 	2. 	3. 
4. 	5. 	6. 
7. 	8. 	9. 

Match a graph from the table above with each of the descriptions given below. Write the number of the graph after each description.

- a) The graph of $f(x) = 2 \cdot \left(\frac{1}{3}\right)^x$ (9) (2)
- b) The graph $y = \frac{4}{x-1} + 2$ (2) (2)
- c) The graph of $g(x) = ax^2 + bx + c$ with $a > 0$ and $c < 0$ (5) (2)
- d) The graph $y = (x + 5)^2 - 2$ (4) (2)

Question 4:

Consider the following quadratic function:

$$f(x) = -4x^2 + 8x + 2$$

- a) Solve the equation $0 = -4x^2 + 8x + 2$ by completing the square. (4)

$$x = 2.22, x = -0.225$$

- b) Determine the coordinates of the vertex of $f(x)$. (2)

$$(1, 6)$$

- c) The graph of $f(x)$ is transformed by translating the graph three units to the left and one unit up. Write down the coordinates of the vertex for this new graph. (2)

$$(-2, 7)$$

Question 5:

Given below are three quadratic equations.

a) $0 = 2x^2 + 7x + 15$ b) $0 = \frac{1}{2}x^2 - 2x + 2$ c) $0 = 2x^2 - 6x - 9$

- a) Which one of the equations will have 2 solutions? Provide some calculation/explanation to support your choice. *No Soluⁿ* *1 soluⁿ* *two Sol(2)ⁿ*

$$\textcircled{a} \Delta = -$$

$$\textcircled{b} \Delta = 0$$

$$\textcircled{c} \Delta = +$$

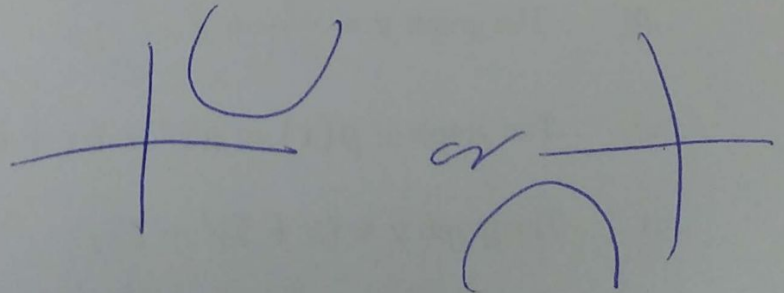
- b) Solve equation c), and give your answer to 3 significant figures, if it exists. (3)

$$x = 1.10$$

$$x = 4.10$$

- c) Explain, in words, what it means when a quadratic equation has non-real roots? You can use a diagram to accompany your explanation. (2)

The graph does not have ~~x~~-intercepts



Question 6:

Solve the following by substitution

(4)

$$-x^2 - x + 19 = y$$

$$x = y + 80$$

Question 7:

A man jumps from an airplane and his speed, V , of descent is given by

$$V = 50(1 + 2^{0.5t}) \text{ m/s, where } t \text{ is the time in seconds.}$$

a) What was the speed of the man 2 seconds after he jumped?

(2)

50

b) Find the time taken for his speed to reach 450m/s.

(3)

6 sec

Question 8:

Solve the following equations without using a calculator.

(a) $\log_x 49 = 2$

$x^2 = 49 \Rightarrow x = \pm 7$

(3)

(b) $\log_2 8 = x$

$x = 3$

(2)

(c) $\log_{25} x = -\frac{1}{2}$

$x = \frac{1}{5}$

(3)

1. Solve $\log_2 x + \log_2(x-2) = 3$, for $x > 2$.

$$\begin{aligned} x(x-2) &= 2^3 & (7) \\ x^2 - 2x - 8 &= 0 \\ x &= 4 & x = -2 \end{aligned}$$

2. Consider the arithmetic sequence 3, 9, 15, ..., 1353.

(a) Write down the common difference.

$$d = 6 \quad (1)$$

(b) Find the number of terms in the sequence.

$$\begin{aligned} U_n &= U_1 + (n-1)d \\ \frac{1353 - 3}{6} + 1 &= n & \Rightarrow n = 225 & (3) \end{aligned}$$

3. Arturo goes swimming every week. He swims 200 metres in the first week. Each week he swims 30 metres more than the previous week. He continues for one year (52 weeks).

(a) How far does Arturo swim in the final week?

$$\begin{aligned} U_{52} &= U_1 + (n-1)d & (3) \\ &= 200 + (52-1)30 \\ &= 1730 \end{aligned}$$

4. A theatre has 20 rows of seats. There are 15 seats in the first row, 17 seats in the second row, and each successive row of seats has two more seats in it than the previous row.

(a) Calculate the number of seats in the 20th row.

$$U_{20} = 15 + (20-1)(2) = 53$$

5. Consider the arithmetic sequence 2, 5, 8, 11,

(a) Find u_{101} . $= 2 + (101-1)(3) = 302$ (3)

(b) Find the value of n so that $u_n = 152$. (3)

$$\begin{aligned} 152 &= 2 + (n-1)(3) \\ 51 &= n \end{aligned}$$

7. Let $f(x) = 3x$, $g(x) = 2x - 5$ and $h(x) = (f \circ g)(x)$.

(a) Find $h(x)$. $= 3(2x - 5) = 6x - 15$

(2)

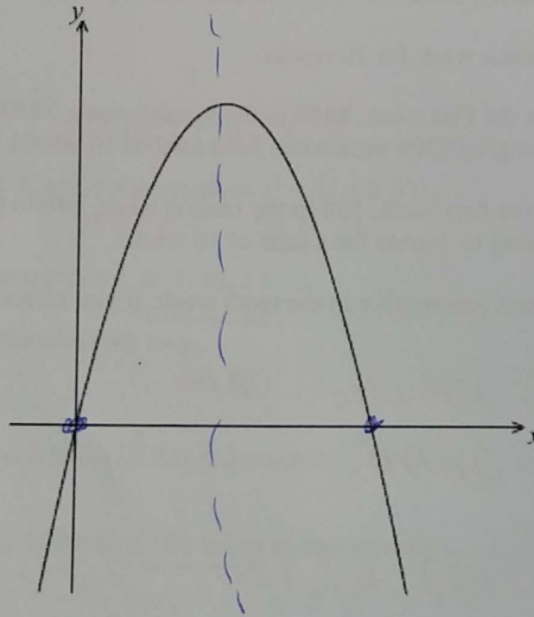
(b) Find $h^{-1}(x)$.

$$x = 6y - 15$$

$$y = \frac{x + 15}{6}$$

(3)

8. Let $f(x) = 8x - 2x^2$. Part of the graph of f is shown below.



(a) Find the x -intercepts of the graph.

$$x = 0, x = 4$$

(4)

(b) (i) Write down the equation of the axis of symmetry.

$$x = 2$$

(1)

(ii) Find the y -coordinate of the vertex.

$$\begin{aligned} \text{E.g. } y &= 8(2) - 2(2)^2 \\ &= 8 \end{aligned}$$

(2)

1. A scientist earns an annual salary of 45 000 USD for the first year of her employment. Her annual salary increases by 1750 USD each year.

(a) Calculate the annual salary for the fifth year of her employment. (3)

$$U_5 = 45000 + (5-1)1750 = 52000$$

She remains in this employment for 10 years.

(b) Calculate the **total** salary she earns in this employment during these 10 years. (3)

$$S_{10} = \frac{(U_1 + U_{10})n}{2} = \frac{(45000 + 60750)10}{2} = 528750$$

$$U_{10} = 45000 + 9(1750) = 60750$$

A National Lottery is offering prizes in a new competition. The winner may choose:

Option one: \$1000 each week for 10 weeks.

Option two: \$250 in the first week, \$450 in the second week, \$650 in the third week, increasing by \$200 each week for a total of 10 weeks.

Option three: \$10 in the first week, \$20 in the second week, \$40 in the third week continuing to double for a total of 10 weeks.

(a) Calculate the amount you receive in the tenth week, if you select

(i) **option two;** (3)

$$U_{10} = 250 + (10-1)200 = 2050$$

(ii) **option three.** (3)

$$U_{10} = 10(2)^9 = 5120$$

(b) What is the **total** amount you receive if you select **option two**? (2)

$$S_{10} = \frac{(U_1 + U_{10})n}{2} = \frac{(250 + 2050)10}{2} = 11500$$

(c) Which option has the greatest total value? Justify your answer by showing all appropriate calculations. (4)

option 1

$$S_{10} = 1000 \times 10 = 10,000$$

option 2

$$S_{10} = 11500$$

option 3

$$S_{10} = \frac{10(2^{10}-1)}{2-1} = 10230$$

↑
Best

3. A tree begins losing its leaves in October. The number of leaves that the tree loses each day increases by the same number on each successive day.

Date in October	1	2	3	4
Number of leaves lost	24	40	56	72

(a) Calculate the number of leaves that the tree loses on the 21st October. (3)

$$U_n = U_1 + (n-1)d = 24 + (21-1)16 = 344$$

(b) Find the total number of leaves that the tree loses in the 31 days of the month of October. (3)

$$S_{31} = \frac{2(U_1 + (n-d))}{2} = \frac{2(24 + 544)}{2} = 568$$

4. Let $f(x) = x^2 - 6x + 8$.

(a) Factorize $x^2 - 6x + 8$. (2)

$$(x-4)(x-2)$$

(b) Hence, or otherwise, solve the equation $x^2 - 6x + 8 = 0$. (2)

$$x=4 \quad x=2$$

5. Consider the arithmetic sequence 1, 4, 7, 10, 13, ...

(a) Find the value of the eleventh term. (2)

$$U_{11} = 31$$

(b) The sum of the first n terms of this sequence is $\frac{n}{2}(3n-1)$. (2)

(i) Find the sum of the first 100 terms in this arithmetic sequence. (2)

$$S_{100} = \frac{100}{2}(3(100)-1) = 14950$$

(ii) The sum of the first n terms is 477. (2)

$$3n^2 - n - 954 = 0 \Rightarrow \frac{n}{2}(3n-1) = 477 \times 2$$

$$3n^2 - n - 954 = 0$$

6. The seventh term, u_7 , of a geometric sequence is 108. The eighth term, u_8 , of the sequence is 36. (2)

(a) Write down the common ratio of the sequence. (1)

$$r = \frac{u_8}{u_7} = \frac{36}{108} = \frac{1}{3}$$

(b) Find u_1 . (2)

$$108 = u_1 \left(\frac{1}{3}\right)^{7-1} \Rightarrow u_1 = 78732$$

The sum of the first k terms in the sequence is 118 096. (3)

(c) Find the value of k . (3)

$$S_k = \frac{u_1(r^k - 1)}{r - 1} = \frac{78732 \left(\frac{1}{3}^k - 1\right)}{\frac{1}{3} - 1} = 118096$$

$$k = 10$$

7. In 2000 Herman joined a tennis club. The fees were £1200 a year. Each year the fees increase by 3%.

$$r = \frac{103}{100} = 1.03$$

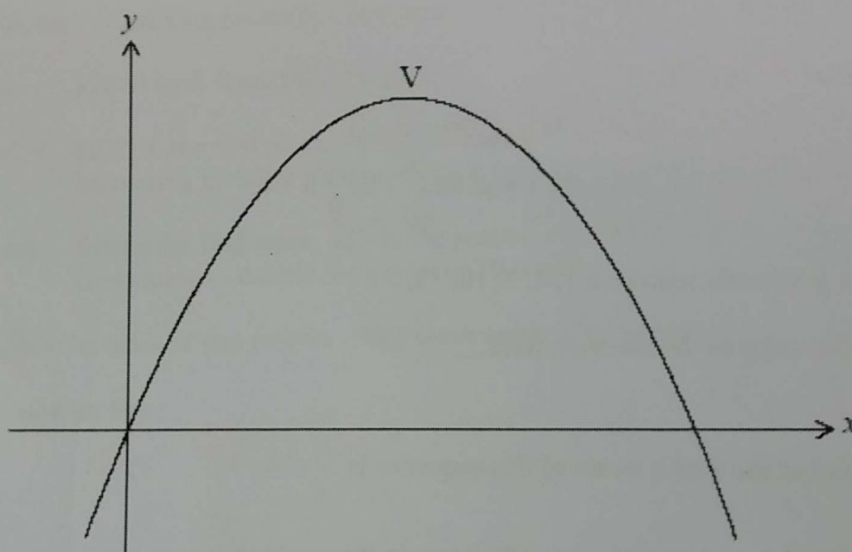
- (a) Calculate, to the nearest £1, the fees in 2002. (4)

$$U_2 = U_1 r^{n-1} \Rightarrow 1200 (1.03)^{2-1} = 1236$$

- (b) Calculate the total fees for Herman who joined the tennis club in 2000 and remained a member for five years. (4)

$$S_{n-1} = U_1 r^{n-1} = 1200 (1.03)^{5-1} = 1350 \rightarrow 1351$$

8. A quadratic curve with equation $y = ax(x - b)$ is shown in the following diagram.



The x -intercepts are at $(0, 0)$ and $(6, 0)$, and the vertex V is at $(h, 8)$.

- (a) Find the value of h . $h = 3$ (2)

- (b) What is the equation of the line of symmetry? (1)

$$x = 3$$

9. In an experiment researchers found that a specific culture of bacteria increases in number according to the formula

$$N = 150 \times 2^t,$$

where N is the number of bacteria present and t is the number of hours since the experiment began.

Use this formula to calculate

- (a) the number of bacteria present at the start of the experiment; **150** (1)
- (b) the number of bacteria present after 3 hours; **1200** (1)
- (c) the number of hours it would take for the number of bacteria to reach 19 200. **$t = 7$ hours.** (2)

10. Find the sum of this arithmetic series:

$$\sum_{n=1}^{25} 3n + 5 = S_{25} = \frac{(2U_1 + (n-1)d) \frac{n}{2}}{2}$$

$$= \frac{[2(8) + (25-1) \times 3] \frac{25}{2}}{2}$$

$U_1 = 8$
 $U_2 = 11$
 $d = 3$
 $= 1100$

11. Find the sum of this geometric series:

$$\sum_{n=1}^{11} 4 \times 2^{n-1}$$

$$S_{11} = \frac{U_1(r^n - 1)}{r - 1}$$

$U_1 = 4$
 $U_2 = 8$
 $r = 2$

$$= \frac{4(2^{11} - 1)}{2 - 1}$$

$$= 8188$$

(4)