Binomial Theorem *[40 marks]*

1a. Expand $(2 + x)^4$ and simplify your result. *[3 marks]*

$1(2)^4(x)^0 + 4(2)^3(x)^1 + 6(2)^2(x)^2 + 4(2)^1(x)^3$
$+ 1(2)^0(x)^4$

$= 16 + 32x + 24x^2 + 8x^3 + x^4$

[3 marks]

1b. Hence, find the term in x^2 in $(2+x)^4\left(1+\frac{1}{x^2}\right)$.

$$(16+32x+24x^2+8x^3+x^4)\left(1+\frac{1}{x^2}\right)$$

1

2

from 1 $24x^2(1) = 24x^2$

from 2 $x^4\left(\frac{1}{x^2}\right) = 1x^2$ +

$\boxed{25x^2}$

2a. The fifth term in the expansion of the binomial $(a+b)^n$ is given by $\binom{10}{4} p^6 (2q)^4$. [1 mark]

Write down the value of n.

$n = 10$

2b. Write down a and b, in terms of p and/or q. [2 marks]

$a = p$

$b = 2q$

[5 marks]

3. a. Expand and simplify $\left(x - \frac{2}{x}\right)^4$.

b. Hence determine the constant term in the expansion $(2x^2 + 1)\left(x - \frac{2}{x}\right)^4$.

(a) $1(x)^4\left(\frac{-2}{x}\right)^0 + 4(x)^3\left(\frac{-2}{x}\right)^1 + 6(x)^2\left(\frac{-2}{x}\right)^2 + 4(x)^1\left(\frac{-2}{x}\right)^3$
$+ 1(x)^0\left(\frac{-2}{x}\right)^4$

$= x^4 - 8x^2 + 24 - \frac{32}{x^2} + \frac{16}{x^4}$

(b) $\left(x^4 - 8x^2 + 24 - \frac{32}{x^2} + \frac{16}{x^4}\right)\left(2x^2 + 1\right)$

① $: 24 \cdot (1) = 24$

② $\frac{-32}{x^2} \cdot 2x^2 = -64$ +

-40

4.

[6 marks]

Consider the expansion of $\left(3x^2 - \frac{1}{x}\right)^9$.

(a) How many terms are there in this expansion?

(b) Find the constant term in this expansion.

(a) # terms = 10 [redacted]

$n+1 = 9+1 = 10$

(b) ~~$T_{r+1} = 3x^2$~~ $T_{r+1} = \binom{n}{r}(3x^2)^{n-r}\left(\frac{-1}{x}\right)^r$

$= \binom{9}{r}(3x^2)^{n-r}\left(\frac{-1}{x}\right)^r$

$= \binom{9}{r} 3^{n-r} x^{2(n-r)} \left(\frac{-1^n}{x^{+r}}\right)$

$\binom{9}{r} 3^{n-r} \underbrace{x^{2n-2r} x^{-r}}_{x^{\boxed{2n-3r}} = 0} (-1)^n$

$2n - 3r = 0$

$n=9$ $\quad 18 - 3r = 0$

$\boxed{r = 6}$

$\binom{9}{6} 3^{9-6} x^{2(9)-3(6)} (-1)^6$

$= 84(3)^3$

$= 2268$

$\binom{9}{6} = \frac{9 \times 8 \times 7 \times \cancel{6!}}{\cancel{6!}(9-6)!}$

$= \frac{9 \times 8 \times 7}{3 \times 2 \times 1}$

$= 84$

5. Find the coefficient of x^5 in the expansion of $(x+2)(x^2+1)^8$. [5 marks]

$$(x^2+1)^8$$

Find x^4 x^5

$n=8$

$a=x^2$

$b=1$

$T_{r+1} = T_{r+1} = \binom{8}{r}(x^2)^{(8-r)}(1)^r$

$2(8-r)=4$

$16-2r=4$

$r=\cancel{8}6$

$T_{\cancel{8}6+1} = T_{\cancel{9}7} = \binom{8}{6}(x^2)^{8-6}(1)^4$

$$\boxed{=28x^4}$$

$2(8-r)5 = 16-2r=5$

r should be an integer!

The coefficient is 28

$\cancel{\frac{8\times7\times6\times5\times4!}{4!(8-4)!}}$

$\cancel{=\frac{8\times7\times6\times5}{4\times3\times2\times1}}$

$\cancel{=70}$

$\frac{8\times7\times6!}{6!(8-6)!}$

$=\frac{8\times7}{2\times1}=28$

[6 marks]

6. Consider the expression $(x^2y-2y^2)^6$. Find the term in which x and y are raised to the same power.

$T_{r+1} = \binom{6}{r}(x^2y)^{6-r}(-2y^2)^r$

$\binom{6}{r} x^{2(6-r)} y^{6-r} (-2)^r y^{2(r)}$

$\binom{6}{r} x^{12-2r} y^{6-r+2r} (-2)^r$

$\binom{6}{r} x^{(12-2r)} y^{6+r} (-2)^r$

$12-2r = 6+r$

$12 = 6+3r$

$6 = 3r$

$\boxed{r=2}$

$\binom{6}{2} x^{12-2(2)} y^{6+2} (-2)^2$

$=30(-2)^2$

$=\cancel{160}\ x^4 y^4$

60

$\frac{6\times5\times\cancel{4!}}{2!\cancel{(6-2)!}}$

$=\frac{{}^{3}\cancel{6}\times5}{\cancel{2}}=15$

[6 marks]

7. Find the:

a 5th row of Pascal's triangle

b binomial expansion of $\left(x - \frac{2}{x}\right)^5$.

(a)

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

5th row → 1 5 10 10 5 1

(b) $\left(x-\frac{2}{x}\right)^5 = 1(x)^5\left(\frac{-2}{x}\right)^0$

$+5(x)^4\left(\frac{-2}{x}\right)^1 + 10(x)^3\left(\frac{-2}{x}\right)^2$

$+10(x)^2\left(\frac{-2}{x}\right)^3 + 5(x)^1\left(\frac{-2}{x}\right)^4$

$+1(x)^0\left(\frac{-2}{x}\right)^5$

$= x^5 + -10x^3 + 40x$

$-\frac{80}{x} + \frac{80}{x^3} + \frac{-32}{x^5}$