



Mathematics Department

Grade 10 B

Summative Assessment

Thursday January 5, 2017

Name: _____

Answer Key

Instructions:

1. Answer all the questions in the spaces provided.
2. Show ALL the working out required to get answers.
3. A graphic display calculator is allowed for this paper.
4. Unless stated otherwise, give all answers to **3 significant figures**.
5. The total marks for this paper is 54.

Question 1: [5 marks]

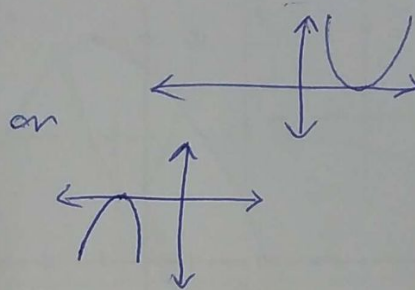
Consider the following quadratic function:

$$f(x) = 3x^2 - 2x + c$$

- a) What should the value of c be so that $f(x)$ has only one root. (3)

$$\begin{aligned} a=3, b=-2, c=c \\ b^2 - 4ac = 0 & \quad | \quad c = \frac{1}{3} \\ (-2)^2 - 4(3)c = 0 & \end{aligned}$$

- b) Sketch a basic graph of any quadratic function that has only one root. (2)



Question 2: [4 marks]

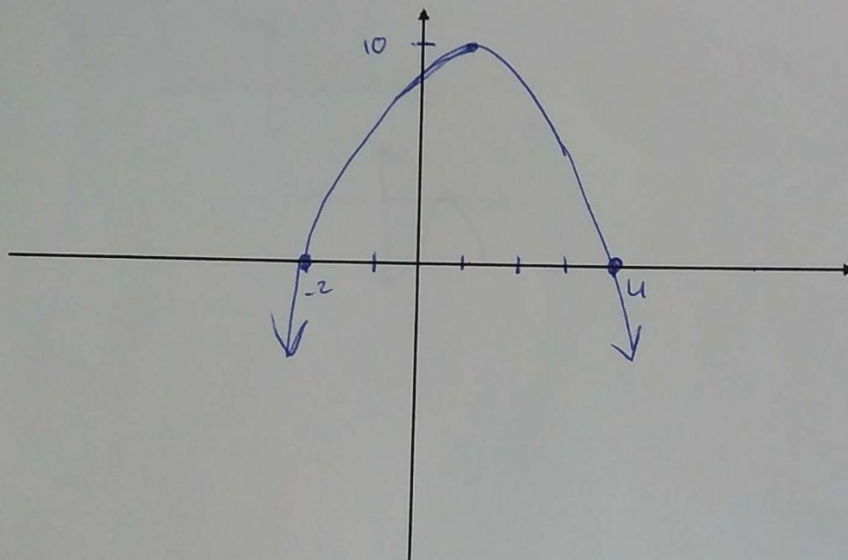
A quadratic function $f(x) = ax^2 + bx + c$ has the following features:

$a < 0$ and $c > 0$

$(-2, 0)$ $(4, 0)$ $(1, 10)$

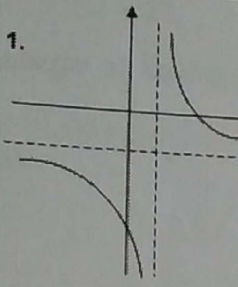
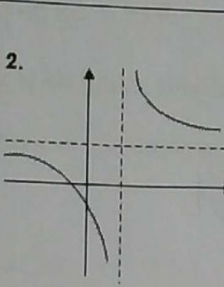
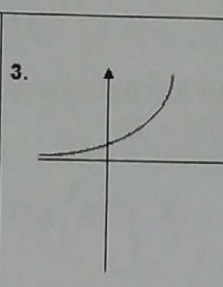
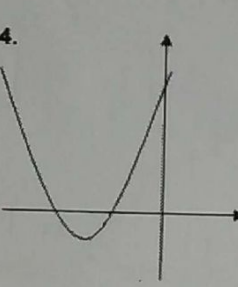
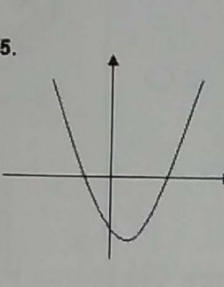
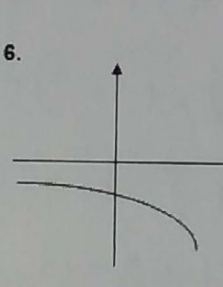
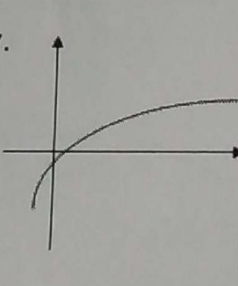
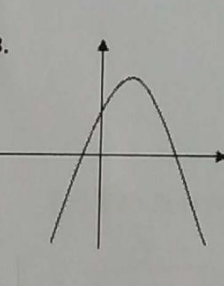
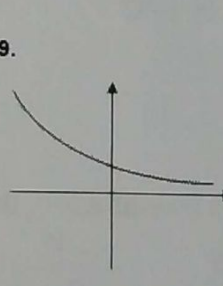
Sketch the graph of $f(x)$, and only include the features mentioned above.

(4)



Question 3: [8 marks]

The table below shows graphs of 9 different functions.

<p>1.</p> 	<p>2.</p> 	<p>3.</p> 
<p>4.</p> 	<p>5.</p> 	<p>6.</p> 
<p>7.</p> 	<p>8.</p> 	<p>9.</p> 

Match a graph from the table above with each of the descriptions given below. Write the number of the graph after each description.

- a) The graph of $f(x) = 2 \cdot \left(\frac{1}{3}\right)^x$ = 9 (2)
- b) The graph $y = \frac{4}{x-1} + 2$ = 2 (2)
- c) The graph of $g(x) = ax^2 + bx + c$ with $a > 0$ and $c < 0$, = 5 (2)
- d) The graph $y = (x + 5)^2 - 2$ = 4 (2)

Question 4: [9 marks]

Consider the following quadratic function:

$$f(x) = -4x^2 + 8x + 2$$

- a) What feature of this function would you calculate if you solved the equation $0 = -4x^2 + 8x + 2$? (1)

X-intercepts

- b) Solve the equation $0 = -4x^2 + 8x + 2$ by completing the square. (4)

$$-4(x^2 - 2x) + 2 = 0$$

$$-4(x^2 - 2x + 1 - 1) + 2 = 0$$

$$-4(x-1)^2 + 4 + 2 = 0$$

$$-4(x-1)^2 + 6 = 0$$

$$(x-1)^2 = \frac{-6}{-4}$$

$$\left(\frac{-2}{2}\right)^2 = 1$$

$$(x-1)^2 = \frac{3}{2}$$

$$x = \pm \sqrt{\frac{3}{2}} + 1$$

$$x = \sqrt{\frac{3}{2}} + 1 \quad x = 1 - \sqrt{\frac{3}{2}}$$

- c) Determine the coordinates of the vertex of $f(x)$. (2)

$$V: \Rightarrow x_{\text{Coor}} = \frac{-b}{2a} = \frac{-8}{2(-4)} = +1$$

$$y_{\text{Coor}} = -4(1)^2 + 8(-1) + 2$$

$$V: (-1, 6)$$

- d) The graph of $f(x)$ is transformed by translating the graph three units to the left and one unit up. Write down the coordinates of the vertex for this new graph. (2)

$$(-4, 7)$$

$$\begin{matrix} -3 \\ +1 \end{matrix}$$

Question 5: [7 marks]

Given below are three quadratic equations.

a) $0 = 2x^2 + 7x + 15$ b) $0 = \frac{1}{2}x^2 - 2x + 2$ c) $0 = 2x^2 - 6x - 9$

a) Which one of the equations will have 2 solutions? Provide some calculation/explanation to support your choice. (2)

Ⓐ $\Delta = b^2 - 4ac$
 $= 7^2 - 4(2)(15)$
 $= -71 < 0$
 No Soluⁿ

Ⓑ $\Delta = b^2 - 4ac$
 $= (-2)^2 - 4(\frac{1}{2})(2)$
 $= 0$
 1-Soluⁿ

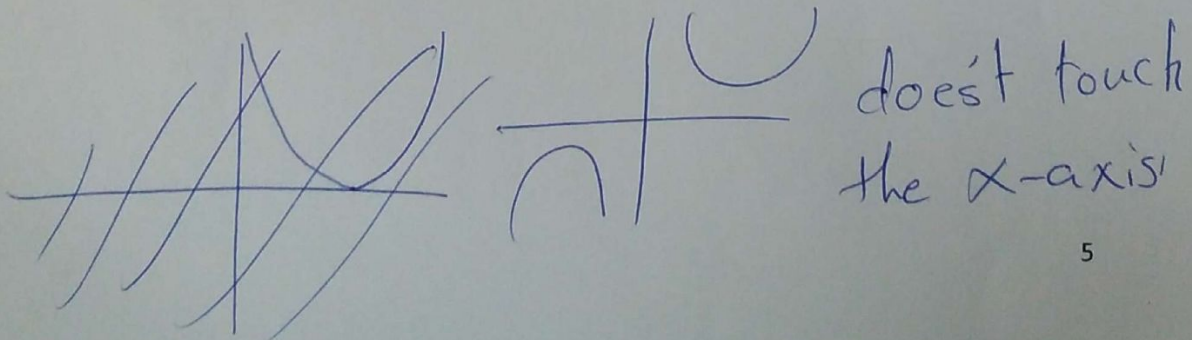
Ⓒ $\Delta = b^2 - 4ac$
 $= (-6)^2 - 4(2)(-9)$
 $= 108 > 0$
 2 Soluⁿ ✓

b) Solve equation (C) and give your answer to 3 significant figures if they exist. (3)

$$x = \frac{-(-6) \pm \sqrt{108}}{2(2)}$$

~~...~~
 $x = 4.10$ $x = -1.10$

c) Explain, in words, what it means when a quadratic equation has non-real roots? You can use a diagram to accompany your explanation. (2)



Question 6 [4 marks]

It is given that $2^{2x} - 2^x - 12$ can be written as $(2^x + a)(2^x + b)$.

- a) Determine the values of a and b .

$$y = 2^x$$

$$y^2 - y - 12$$

$$(y-4)(y+3)$$

$$(2^x - 4)(2^x + 3)$$

$$a = -4$$

$$b = 3$$

(2)

- b) Hence, determine all **valid** solutions to the equation $2^{2x} - 2^x - 12 = 0$.

(2)

$$(y-4)(y+3) = 0$$

$$(2^x - 4)(2^x + 3) = 0$$

$$2^x = 4$$

$$2^x = -3$$

$$2^x = 2^2$$

No' Soluⁿ

$$x = 2$$

Question 7 [5 marks]

A man jumps from an airplane and his speed, V , of descent is given by $V = 50(1 + 2^{0.5t})$ m/s where t is the time in seconds.

- a) What was the speed of the man 2 seconds after he jumped? (2)

$$\begin{aligned} V &= 50(1 + 2^{0.5(2)}) \\ &= 50(1 + 2^1) \\ &= 150 \end{aligned}$$

- b) Find the time taken for his speed to reach 450m/s. (3)

$$\begin{aligned} \frac{450}{50} &= \frac{50}{50} (1 + 2^{0.5t}) \\ 9 &= 1 + 2^{0.5t} \\ 8 &= 2^{0.5t} \\ 2^3 &= 2^{0.5t} \\ 0.5t &= 3 \\ t &= 6 \text{ Sec.} \end{aligned}$$

Question 8 [12 marks]

1) Solve the following equations without using a calculator.

(a) $\log_x 49 = 2$

(3)

$$x^2 = 49$$

$$x^2 = 7^2$$

$$\boxed{x = 7}$$

(b) $\log_2 8 = x$

(2)

$$2^x = 8$$

$$2^x = 2^3$$

$$\boxed{x = 3}$$

(c) $\log_{25} x = -\frac{1}{2}$

(3)

$$25^{-\frac{1}{2}} = x$$

$$\frac{1}{25^{\frac{1}{2}}} = x$$

$$x = \frac{1}{\sqrt{25}}$$

$$x = \frac{1}{5}$$

2) Solve the equation $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$.

(4)

$$3^{2(x-1)} = 3^{-2x}$$

$$\frac{2(x-1)}{2} = \frac{-2x}{2}$$

$$\begin{array}{r} x-1 \\ +x \end{array} = \begin{array}{r} -x \\ +x \end{array}$$

$$2x = 1$$

$$\boxed{x = \frac{1}{2}}$$