

Converting Quadratic Equations between Standard and Vertex Form

Standard Form: $y = ax^2 + bx + c$

Vertex Form: $y = a(x - h)^2 + k$

Convert from Standard Form to Vertex Form:

$$y = ax^2 + bx + c \quad \Longleftrightarrow \quad y = a(x - h)^2 + k$$

know a, b, c want a, h, k

$$a = a$$

$$x = \frac{-b}{2a} = h$$

Solve for $y = k$

Substitute the values and rewrite.

Example 1:

$y = 8x^2 - 16x + 27$

$a = 8$

$h = x = \frac{-b}{2a} = \frac{-(-16)}{2(8)} = \frac{16}{16} = 1$

$k = y = 8(1)^2 - 16(1) + 27 = 8 - 16 + 27 = 19$

$y = 8(x - 1)^2 + 19$

We know a, b, c and want a, h, k

← a is the coefficient of the x^2 term

← use the formula to find the value of h

← substitute the value found for h into the original equation and solve for k

Example 2:

$y = 5x^2 - 40x + 67$

$a = 5$

$h = x = \frac{-b}{2a} = \frac{-(-40)}{2(5)} = \frac{40}{10} = 4$

$k = y = 5(4)^2 - 40(4) + 67 = 80 - 160 + 67 = -13$

$y = 5(x - 4)^2 - 13$

We know a, b, c and want a, h, k

← a is the coefficient of the x^2 term

← use the formula to find the value of h

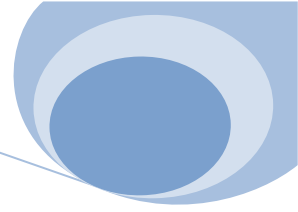
← substitute the value found for h into the original equation and solve for k

Practice: Convert the following quadratics from standard to vertex form.

1. $y = 5x^2 - 10x + 37$

2. $y = 7x^2 + 28x + 19$

3. $y = -2x^2 - 24x - 75$



Convert from Vertex Form to Standard Form:

$$y = a(x - h)^2 + k \quad \Rightarrow \quad y = ax^2 + bx + c$$

Example 1:

$$\begin{array}{l} \mathbf{y = 5(x + 2)^2 - 9} \\ y = 5(x + 2)(x + 2) - 9 \quad \leftarrow \text{Rewrite } (x + 2)^2 \\ y = 5(x^2 + 4x + 4) - 9 \quad \leftarrow \text{Simplify } (x + 2)(x + 2) \\ y = 5x^2 + 20x + 20 - 9 \quad \leftarrow \text{Distribute the 5} \\ \mathbf{y = 5x^2 + 20x + 11} \quad \leftarrow \text{Combine Like Terms} \end{array}$$

Example 2:

$$\begin{array}{l} \mathbf{y = -3(x - 4)^2 + 7} \\ y = -3(x - 4)(x - 4) + 7 \quad \leftarrow \text{Rewrite } (x - 4)^2 \\ y = -3(x^2 - 8x + 16) + 7 \quad \leftarrow \text{Simplify } (x - 4)(x - 4) \\ y = -3x^2 + 24x - 48 + 7 \quad \leftarrow \text{Distribute the } -3 \\ \mathbf{y = -3x^2 + 24x - 41} \quad \leftarrow \text{Combine Like Terms} \end{array}$$

Practice: Convert the following quadratics from vertex to standard form.

1. $y = (x - 2)^2 + 6$

2. $y = 3(x - 3)^2 - 12$

3. $y = -2(x + 1)^2 + 3$