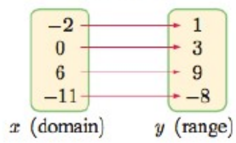
Review:

Functions

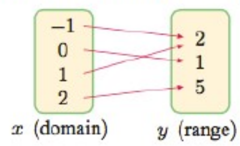
MAPPINGS

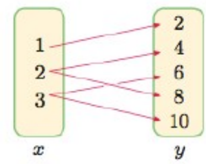
A **mapping** is used to map the members or **elements** of one set called the **domain**, onto the members of another set called the **range**.

For the mapping $y = x + 3$:



For the mapping $y = x^2 + 1$



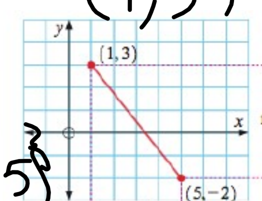


FUNCTIONS

A **function** is a mapping in which each element of the domain maps onto *exactly one* element of the range.

DOMAIN AND RANGE

(1,3)



The **domain** of a relation is the set of possible values that x may have.
The **range** of a relation is the set of possible values that y may have.

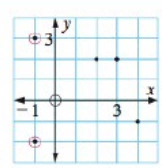
Domain = $\{x \mid 1 \leq x \leq 5\}$

Range = $\{y \mid -2 \leq y \leq 3\}$ (5, −2)

A **function** is a relation in which no two different ordered pairs have the same first member.

So, $\{(-1, 3), (2, 2), (-1, -2), (3, 2), (4, -1)\}$ is not a function.

different ordered pairs with same first member.



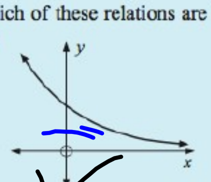
$-1 \to 3$, $-1 \to -2$

GEOMETRIC TEST FOR FUNCTIONS: "VERTICAL LINE TEST"

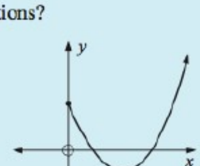
Example 2

Which of these relations are functions?

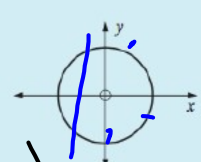
a



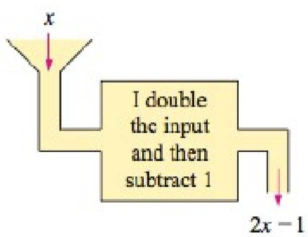
b



c



FUNCTION NOTATION



So, if 3 is fed into the machine, $2(3) - 1 = 5$ comes out.

Example 3

If $f : x \mapsto 3x^2 - 4x$, find the value of: **a** $f(2)$ **b** $f(-5)$

$$f(2) = 3(2)^2 - 4(2)$$

THE DOMAIN OF A FUNCTION

To find the domain of a function, we need to consider what values of the variable make the function undefined.

For example, notice that for:

- $f(x) = \sqrt{x}$, the domain is $\{x \mid x \geqslant 0, x \in \mathbb{R}\}$ since $\sqrt{x}$ has meaning only when $x \geqslant 0$.
- $f(x) = \dfrac{1}{\sqrt{x-1}}$, the domain is $\{x \mid x > 1, x \in \mathbb{R}\}$ since, when $x - 1 = 0$ we are 'dividing by zero', and when $x - 1 < 0$, $\sqrt{x-1}$ is undefined as we can't find the square root of a negative in the real number system.

$f(x) = \sqrt{x}$

$x \geqslant 0$

Domain: $\{x \mid x \geqslant 0, x \in \mathbb{R}\}$

$x + 1 \geqslant 0$

$-1 \quad -1$

$x \geqslant -1$

Domain $\{x \mid x \geqslant -1\}$

Ex 2

$f(x) = \dfrac{1}{\sqrt{x-1}}$

$\sqrt{x-1} \neq 0$

$(\sqrt{x-1})^2 \neq (0)^2$

$x - 1 \neq 0$

$x \neq +1$

$x - 1 \geqslant 0$

$+1 \quad +1$

$x \geqslant +1$

$x > +1$

1 Find the domain of the following:

a $f(x) = \sqrt{x-2}$

b $f(x) = \sqrt{3-x}$

c $f(x) = \sqrt{x} + \sqrt{2-x}$

d $f(x) = \dfrac{1}{\sqrt{x}}$

e $f(x) = \dfrac{1}{\sqrt{x}} + \dfrac{1}{\sqrt{x+2}}$

f $f(x) = \dfrac{1}{x\sqrt{4-x}}$