

Answer Key



Mathematics Department

SL Math Year 1

Sequences, Series and Exponents-Test

Summative Assessment

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Mark: 

Instructions:

1. Do not open the assessment until instructed to so.
2. Answer all questions on the paper (Request extra paper if necessary)
2. Show ALL the working out required to get answers.
3. A graphic display calculator is allowed for this paper

Multiple Choice

Identify the choice that best completes the statement or answers the question.

- D 1. Find the arithmetic mean a_n of $a_{n-1} = 3.9$ and $a_{n+1} = 7.1$.
a. 11 b. 1.6 c. 3.7 d. 5.5

Write the explicit formula for the geometric sequence. Then find the fifth term in the sequence.

- D 2. $a_1 = -4, a_2 = 8, a_3 = -16$
a. $a_n = -2 \cdot (-4)^{n-1}; -512$ c. $a_n = -4 \cdot (-2)^n; 128$
b. $a_n = -4 \cdot (2)^n; -64$ d. $a_n = -4 \cdot (-2)^{n-1}; -64$

What is the sum of the finite arithmetic series?

- B 3. $(-5) + 0 + 5 + 10 + \dots + 65$
a. 455 b. 450 c. 900 d. 445

Does the infinite geometric series diverge or converge? Explain.

- A 4. $\frac{1}{5} + \frac{1}{10} + \frac{1}{20} + \frac{1}{40} + \dots$
a. It converges; it has a sum.
b. It diverges; it does not have a sum.
c. It diverges; it has a sum.
d. It converges; it does not have a sum.

[Total 4 Marks]

SECTION A - SEQUENCES AND SERIES

1. Find k given the consecutive arithmetic terms:

• $5, k, k^2 - 8$

2
[3 Marks]

$$\begin{aligned}
 k - 5 &= k^2 - 8 - k \\
 k^2 - 2k - 3 &= 0 \\
 (k - 3)(k + 1) &= 0 \\
 k &= 3, \quad k = -1
 \end{aligned}$$

2. Find k given that the following are consecutive terms of a geometric sequence:

• $k, k + 8, 9k$

3
[4 Marks]

$$\begin{aligned}
 \frac{k + 8}{k} &= \frac{9k}{k + 8} && \Rightarrow k = -2, \quad k = 4 \\
 (k + 8)^2 &= 9k^2 \\
 k^2 + 16k + 64 &= 9k^2 \\
 8k^2 - 16k - 64 &= 0 \\
 8(k^2 - 2k - 8) &= 0 \\
 8(k + 2)(k - 4) &= 0
 \end{aligned}$$

3. Three consecutive terms of an arithmetic sequence have a sum of 12 and a product of -80 . Find the terms.

$$(x-d), (x), (x+d)$$

[5 Marks]

$$\begin{aligned} x-d + x + x+d &= 12 & \{ & d^2 = 36 \\ 3x &= 12 & | & d = \pm 6 \\ \boxed{x = 4} & & | & \text{Terms are:} \\ (4-d)(4)(4+d) &= -80 & | & x+d = 4+6 = \textcircled{10} \\ 4(4^2-d^2) &= -80 & | & x = \textcircled{4} \\ 16-d^2 &= -20 & | & x-d = 4-6 = \textcircled{-2} \end{aligned}$$

4. Find the sum of the infinite geometric series

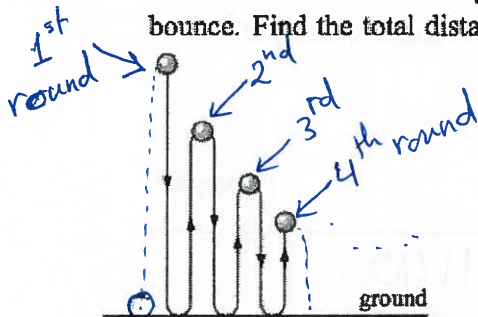
$\rightarrow \infty$

$$\frac{2}{3} - \frac{4}{9} + \frac{8}{27} - \frac{16}{81} + \dots$$

[4 Marks]

$$\begin{aligned} u_1 &= \frac{2}{3} \\ r &= \frac{-2}{3} = \frac{-4/9}{2/3} \\ S_{\infty} &= \frac{u_1}{1-r} = \frac{2/3}{1-(-2/3)} = \frac{2}{5} \end{aligned}$$

5. A ball bounces from a height of 3 metres and returns to 80% of its previous height on each bounce. Find the total distance travelled by the ball until it stops bouncing.



Note: This diagram is inaccurate as the motion is really up and down on the same spot. It has been separated out to help us visualise what is happening.

[7 Marks]

If the ball starts from the bottom, It will move 6 meters (3 up & 3 down). The 2nd round will be 6 meters $\times 0.8 = 6(0.8)$
 3rd round 6 meters $\times 0.8 \times 0.8 = 6(0.8)^2$
 4th " 6 $\times 0.8 \times 0.8 \times 0.8 = 6(0.8)^3$

$\therefore U_1 = 6, r = 0.8$

Total distance = $S_{\infty} - 3 = \frac{6}{1-0.8} - 3 = 27 \text{ meters}$

Full Solution

"We assumed the ball starts from the bottom but if it actually starts from the top, so we have to take out 3 meters."

6. A company offers its employees a choice of two salary schemes A and B over a period of 10 years.

Scheme A: offers a starting salary of \$11000 in the first year and then an annual increase of \$400 per year.

- (a) (i) Write down the salary paid in the second year and in the third year.
 (ii) Calculate the total (amount of) salary paid over ten years.

[3 Marks]

(a) i | $2^{\text{nd}} \text{ yr} = 11000 + 400 = \11400
 $3^{\text{rd}} \text{ yr} = 11000 + 400 + 400 = \11800

ii | $S_{10} = \frac{n}{2} (2u_1 + (n-1)d)$
 $= \frac{10}{2} (2(11000) + 9(400))$
 $= \$128000$

Scheme B: offers a starting salary of \$10000 dollars in the first year and then an annual increase of 7% of the previous year's salary.

- (b) (i) Write down the salary paid in the second year and in the third year.
 (ii) Calculate the salary paid in the tenth year.

[4 Marks]

b i | $2^{\text{nd}} \text{ yr}:$
 $U_2 = 10000(1+0.07) = \$10700$
 $U_3 = 10000(1+0.07)^2 = \11449

ii | $U_{10} = 10000(1+0.07)^9 = 18384.6$
 ≈ 18400

SECTION C - EXPONENTIAL

1. Write as a single power of 3:

a $\frac{27}{9^a} = \frac{3^3}{3^{2a}} = 3^{3-2a}$

b $(\sqrt{3})^{1-x} \times 9^{1-2x}$

[Total 4 Marks]

	$(3)^{\frac{1}{2}(1-x)} \cdot 3^{2(1-2x)}$ $= 3^{\frac{1}{2}(1-x) + 2(1-2x)}$ $= 3^{\frac{1}{2} - \frac{x}{2} + 2 - 4x}$ $= 3^{5/2 - 9/2 x}$
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2. Expand and simplify:

a $e^x(e^{-x} + e^x)$

b $(2^x + 5)^2$

c $(x^{\frac{1}{2}} - 7)(x^{\frac{1}{2}} + 7)$

[Total 6 Marks]

$e^0 + e^{2x}$ $1 + e^{2x}$	$(2^x + 5)(2^x + 5)$ $2^{2x} + 5(2^x) + 5(2^x) + 25$ $2^{2x} + 10(2^x) + 25$	$(x^{\frac{1}{2}})^2 - 7^2$ $= x - 49$
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FOR NON-CRYSTALLINE

$$\rho = \frac{M}{V} = \frac{M}{\frac{M}{\rho_c} + \frac{M}{\rho_a}}$$

$$\rho_c = \frac{M_c}{V_c}$$

total volume

$$V = V_c + V_a$$

$\rho_c = \frac{M_c}{V_c}$	
$\rho_a = \frac{M_a}{V_a}$	
$\rho = \frac{M}{V}$	
$\rho = \frac{M_c + M_a}{V_c + V_a}$	
$\rho = \frac{M_c + M_a}{\frac{M_c}{\rho_c} + \frac{M_a}{\rho_a}}$	
$\rho = \frac{M_c + M_a}{\frac{M_c}{\rho_c} + \frac{M_a}{\rho_a}}$	

FOR CRYSTALLINE

$$\rho_c = \frac{M_c}{V_c}$$

$$\rho_a = \frac{M_a}{V_a}$$

$$\rho = \frac{M}{V}$$

total volume

$\rho_c = \frac{M_c}{V_c}$	$\rho_a = \frac{M_a}{V_a}$	$\rho = \frac{M}{V}$
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