

17

Using a graphic display calculator

CHAPTER OBJECTIVES:

This chapter shows you how to use your graphic display calculator (GDC) to solve the different types of problems that you will meet in your course. You should not work through the whole of the chapter – it is simply here for reference purposes. When you are working on problems in the mathematical chapters, you can refer to this chapter for extra help with your GDC if you need it.

GDC instructions on CD:

The instructions in this chapter are for the TI-Nspire model. Instructions for the same techniques using the TI-84 Plus and the Casio FX-9860GII are available on the CD.



Chapter contents

1 Functions

1.1 Graphing linear functions 572

Finding information about the graph

1.2 Finding a zero 572

1.3 Finding the gradient (slope) of a line 573

Simultaneous equations

1.4 Solving simultaneous equations graphically 574

1.5 Solving simultaneous linear equations 576

Quadratic functions

1.6 Drawing a quadratic graph 577

1.7 Solving quadratic equations 578

1.8 Finding a local minimum or maximum point 579

Exponential functions

1.9 Drawing an exponential graph 583

1.10 Finding a horizontal asymptote 584

Logarithmic functions

1.11 Evaluating logarithms 585

1.12 Finding an inverse function 585

1.13 Drawing a logarithmic graph 588

Trigonometric functions

1.14 Degrees and radians 589

1.15 Drawing trigonometric graphs 590

More complicated functions

1.16 Solving a combined quadratic and exponential equation 591

Modeling

1.17 Using sinusoidal regression 592

1.18 Using transformations to model a quadratic function 594

1.19 Using sliders to model an exponential function 596

2 Differential calculus

Finding gradients, tangents and maximum and minimum points










2.1 Finding the gradient at a point 598

2.2 Drawing a tangent to a curve 599

2.3	Finding maximum and minimum points	600	5.5	Drawing a box and whisker diagram from a list	614
Derivatives			5.6	Drawing a box and whisker diagram from a frequency table	616
2.4	Finding a numerical derivative	602	Calculating statistics		
2.5	Graphing a numerical derivative	603	5.7	Calculating statistics from a list	617
2.6	Using the second derivative	605	5.8	Calculating statistics from a frequency table	618
3	Integral calculus		5.9	Calculating the interquartile range	619
3.1	Finding the value of an indefinite integral	606	5.10	Using statistics	620
3.2	Finding the area under a curve	607	Calculating binomial probabilities		
4	Vectors		5.11	Use of nCr	621
4.1	Calculating a scalar product	608	5.12	Calculating binomial probabilities	622
4.2	Calculating the angle between two vectors	610	Calculating normal probabilities		
5	Statistics and probability		5.13	Calculating normal probabilities from X -values	624
Entering data			5.14	Calculating X -values from normal probabilities	625
5.1	Entering lists of data	612	Scatter diagrams, linear regression and the correlation coefficient		
5.2	Entering data from a frequency table	612	5.15	Scatter diagrams using a Data & Statistics page	627
Drawing charts			5.16	Scatter diagrams using a Graphs page	629
5.3	Drawing a frequency histogram from a list	613			
5.4	Drawing a frequency histogram from a frequency table	614			

Before you start

You should know:


- Important keys on the keyboard: , , , , , , , , 
- The home screen
- Opening new documents, adding new pages, changing settings
- Moving between pages in a document
- Panning and grabbing axes to change a window in a Graphs page
- Change window settings in a Graphs page
- Using zoom tools in a Graphs page
- Using trace in a Graphs page
- Setting the number of significant figures or decimal places

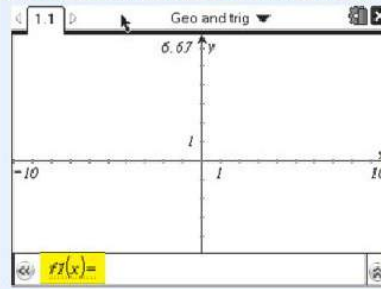
1 Functions

1.1 Graphing linear functions

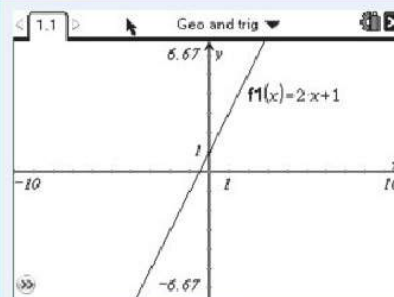
Example 1

Draw the graph of the function $y = 2x + 1$

Open a new document and add a Graphs page.
The entry line is displayed at the bottom of the work area.
The default graph type is Function,
so the form ' $f1(x)=$ ' is displayed.
The default axes are $-10 \leq x \leq 10$ and
 $-6.67 \leq y \leq 6.67$.
Type $2x + 1$ and press .



The graph of $y = 2x + 1$ is now displayed and labeled on the screen.



Finding information about the graph

Your GDC can give you a lot of information about the graph of a function, such as the coordinates of points of interest and the gradient (slope).

1.2 Finding a zero

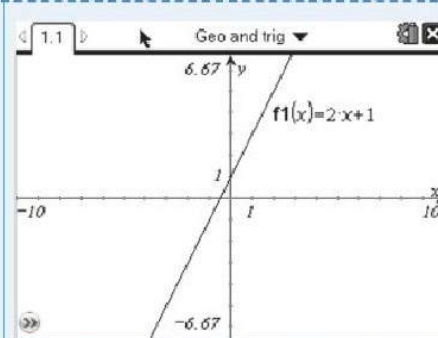
The x -intercept is known as a *zero* of the function.

At the x -intercept, $y = 0$.

Example 2

Find the zero of $y = 2x + 1$

First draw the graph of $y = 2x + 1$ (see Example 1).



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Press **menu** 6:Analyze Graph | 1:Zero

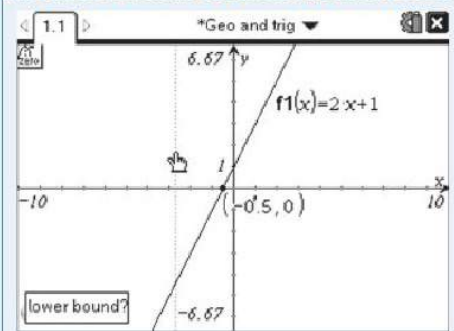
Press **enter**

To find the zero you need to give the lower and upper bounds of a region that includes the zero.

The GDC shows a line and asks you to set the lower bound.

Move the line using the touchpad and choose a position to the left of the zero.

Click the touchpad.

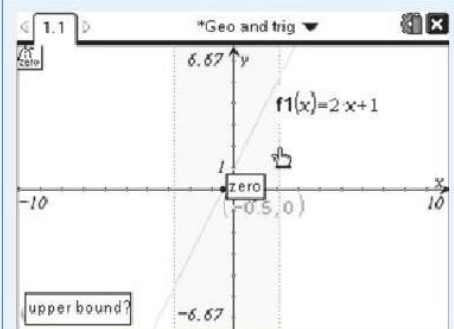


The GDC shows another line and asks you to set the upper bound.

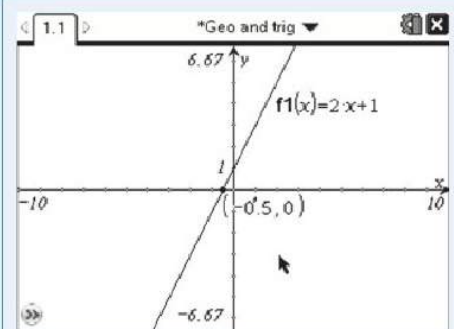
Use the touchpad to move the line so that the region between the upper and lower bounds contains the zero.

When the region contains the zero, the calculator will display the word 'zero' in a box.

Click the touchpad.



The GDC displays the zero of the function $y = 2x + 1$ at the point $(-0.5, 0)$.



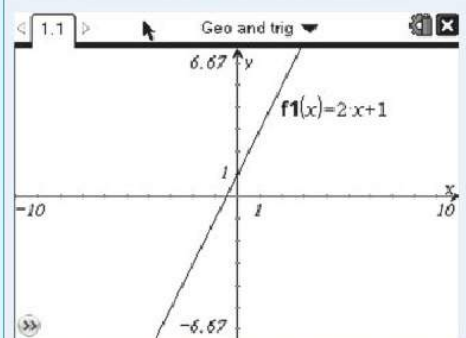
1.3 Finding the gradient (slope) of a line

The correct mathematical notation for gradient (slope) is $\frac{dy}{dx}$, and this is how the GDC denotes gradient.

Example 3

Find the gradient of $y = 2x + 1$

First draw the graph of $y = 2x + 1$ (see Example 1).

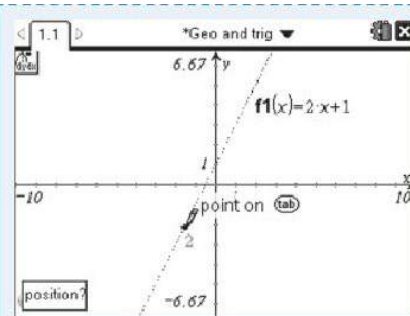


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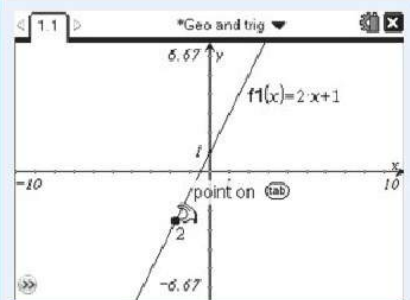
Press **menu** 6:Analyze Graph | 5: $\frac{dy}{dx}$

Press **enter**

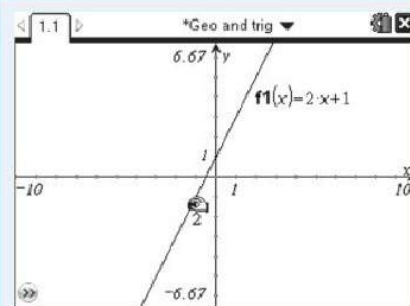
Use the touchpad to select a point on the line.
Click the touchpad.



The point you selected is now displayed together with the gradient of the line at that point.
The gradient (slope) is 2.



With the open-hand symbol showing, click the touchpad again. The hand is now grasping the point.
Move the point along the line using the touchpad.
This confirms that the gradient (slope) of $y = 2x + 1$ at every point on the line is 2.



Simultaneous equations

1.4 Solving simultaneous equations graphically

To solve simultaneous equations graphically you draw the straight lines and then find their point of intersection. The coordinates of the point of intersection give you the solutions x and y .

For solving simultaneous equations using a non-graphical method, see section 1.5.

Example 4

Use a graphical method to solve the simultaneous equations

$$2x + y = 10$$

$$x - y = 2$$

First rewrite both equations in the form ' $y =$ '.

$$2x + y = 10$$

$$y = 10 - 2x$$

$$x - y = 2$$

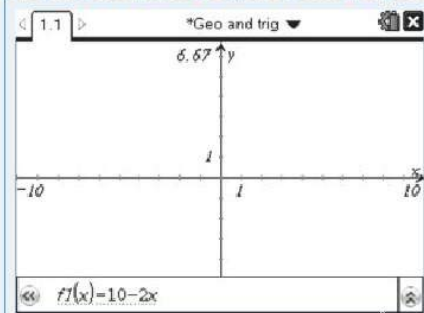
$$-y = 2 - x$$

$$y = x - 2$$

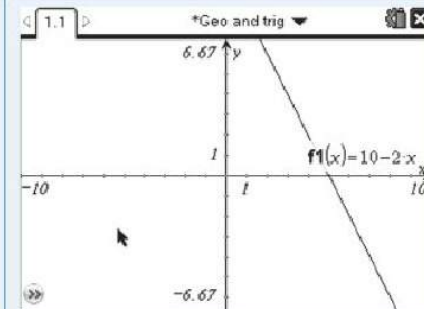
The GDC will only draw the graphs of functions that are expressed explicitly, ' $y =$ ' as a function of x . If the equations are written in a different form, you need to rearrange them before using your GDC to solve them.

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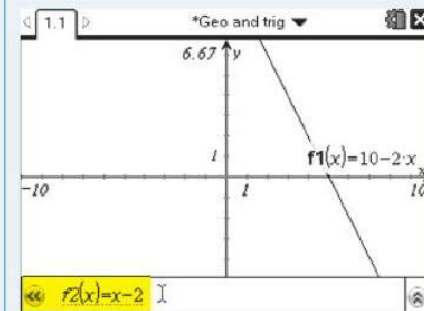
To draw the graphs $y = 10 - 2x$ and $y = x - 2$:
 Open a new document and add a Graphs page.
 The entry line is displayed at the bottom of the work area.
 The default graph type is Function, so the form ' $f1(x)='$ is displayed.
 The default axes are $-10 \leq x \leq 10$ and $-6.67 \leq y \leq 6.67$.



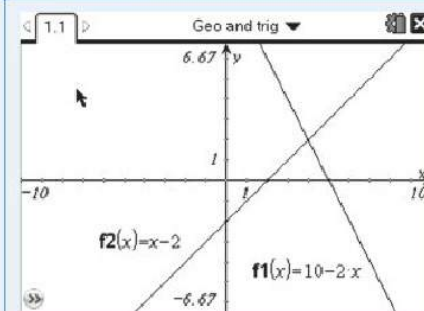
Type $10 - 2x$ and press **enter**.
 The calculator displays the first straight-line graph:
 $f1(x) = 10 - 2x$



Use the touchpad to click on the arrows in the bottom left-hand corner of the screen.
 This will open the entry line again. This time ' $f2(x)='$ is displayed.
 Type $x - 2$ and press **enter**.

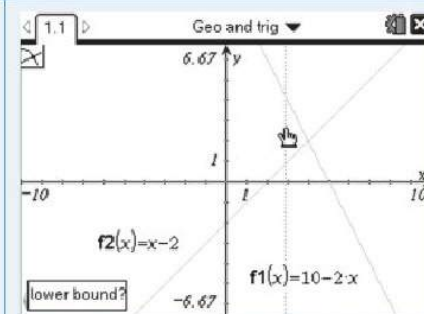


The GDC now displays both straight-line graphs:
 $f1(x) = 10 - 2x$
 $f2(x) = x - 2$



Press **menu** 6:Analyze Graph | 4:Intersection Point(s)
 Press **enter**

To find the intersection you need to give the lower and upper bounds of a region that includes the intersection.
 The GDC shows a line and asks you to set the lower bound. Move the line using the touchpad and choose a position to the left of the intersection.
 Click the touchpad.



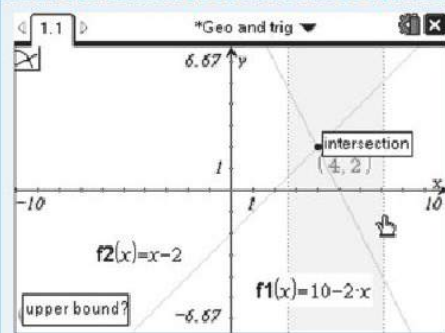
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The GDC shows another line and asks you to set the upper bound.

Use the touchpad to move the line so that the region between the upper and lower bounds contains the intersection.

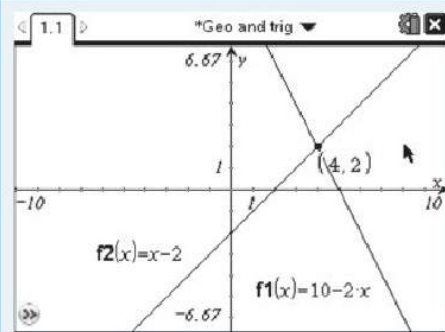
When the region contains the intersection, the calculator will display the word 'intersection' in a box.

Click the touchpad.



The calculator displays the intersection of the two straight lines at the point (4, 2).

The solution is $x = 4$, $y = 2$.



1.5 Solving simultaneous linear equations

When solving simultaneous equations in an examination, you do not need to show any method of solution. You should simply write out the equations in the correct form and then give the solutions. The GDC will do all the working for you.

You do not need the equations to be written in any particular format to use the linear equation solver, as long as they are both *linear*, that is, neither equation contains x^2 or higher order terms.

Example 5

Solve the equations:

$$2x + y = 10$$

$$x - y = 2$$

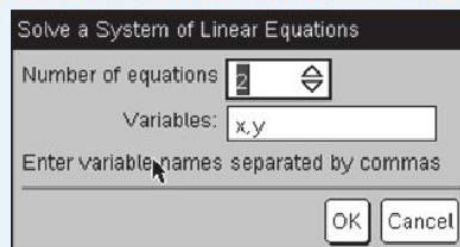
Open a new document and add a Calculator page.

Press **menu** 3:Algebra | 2:Solve Systems of Linear Equations...

Press **enter**

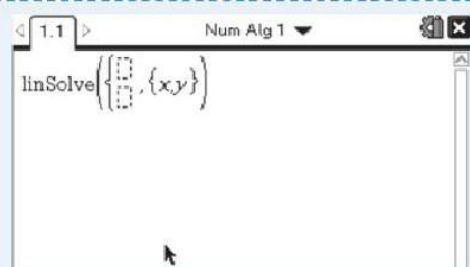
You will see this dialogue box, showing 2 equations and two variables, x and y .

Note: This is how you will use the linear equation solver in your examinations. In your project, you might want to solve a more complicated system with more equations and more variables.

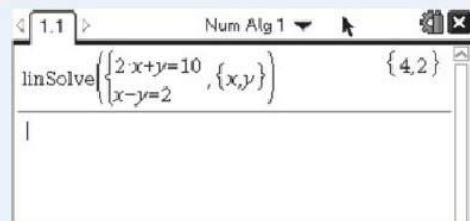


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Press **enter** and you will see the template on the right. Type the two equations into the template, using the arrow keys **▲▼** to move within the template. Press **enter** and the GDC will solve the equations, giving the solutions in the form $\{x, y\}$.



The solutions are $x = 4, y = 2$.



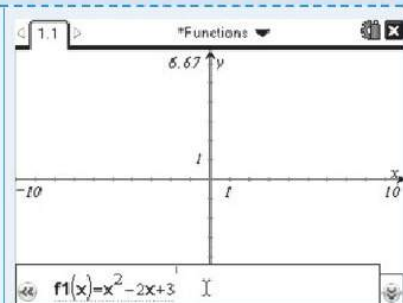
Quadratic functions

1.6 Drawing a quadratic graph

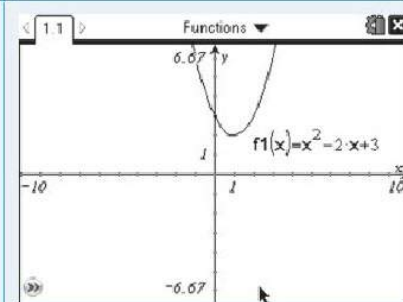
Example 6

Draw the graph of $y = x^2 - 2x + 3$ and display using suitable axes.

Open a new document and add a Graphs page. The entry line is displayed at the bottom of the work area. The default graph type is Function, so the form ' $f1(x) =$ ' is displayed. The default axes are $-10 \leq x \leq 10$ and $-6.67 \leq y \leq 6.67$.

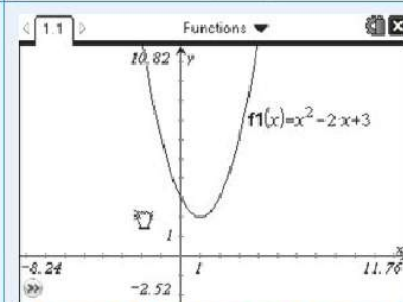


Type $x^2 - 2x + 3$ and press **enter**. The calculator displays the curve with the default axes.



Pan the axes to get a better view of the curve.

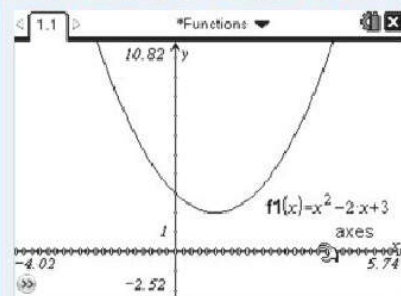
For help with panning, see your GDC manual.



▶ Continued on next page

Grab the x -axis and change it to make the quadratic curve fit the screen better.

For help with changing axes, see your GDC manual.



1.7 Solving quadratic equations

When solving quadratic equations in an examination, you do not need to show any method of solution. You should simply write out the equations in the correct form and then give the solutions. The GDC will do all the working for you.

Example 7

Solve $3x^2 - 4x - 2 = 0$

Press **menu** 3:Algebra | 3:Polynomial Tools | 1:Find Roots of a Polynomial...

Press **enter**

You will see this dialogue box, showing a polynomial of degree 2 (a quadratic equation) with real roots. You do not need to change anything.

Press **enter**

Another dialogue box opens for you to enter the equation. The general form of the quadratic equation is $a_2x^2 + a_1x + a_0 = 0$, so enter the coefficients in a_2 , a_1 and a_0 .

Here, $a_2 = 3$, $a_1 = -4$ and $a_0 = -2$. Be sure to use the **(-)** key to enter the negative values. Use the **tab** key to move around the dialogue box.

Press **enter** and the GDC will solve the equation, giving the roots in the form $\{x, y\}$.

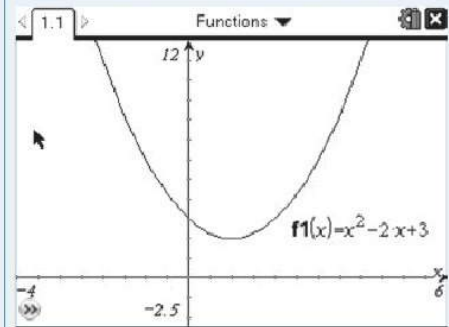
The solutions are $x = -0.387$ or $x = 1.72$ (to 3sf).

1.8 Finding a local minimum or maximum point

Example 8

Find the minimum point on the graph of $y = x^2 - 2x + 3$

First draw the graph of $y = x^2 - 2x + 3$ (see Example 6).



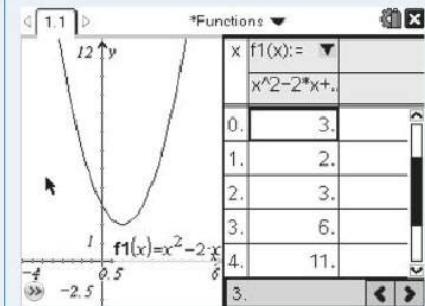
Method 1: Using a table

You can look at the graph **and** a table of the values by using a split screen.

Press **menu** 2:View | 9:Show Table

(or simply press **ctrl** **T**)

The minimum value shown in the table is 2 when $x = 1$.



Look more closely at the values of the function around $x = 1$.

Change the settings in the table.

Choose any cell and press **menu** 5:Table | 5:Edit Table Settings...

Set Table Start to 0.98 and Table Step to 0.01.

Press **enter**

Table

Table Start:

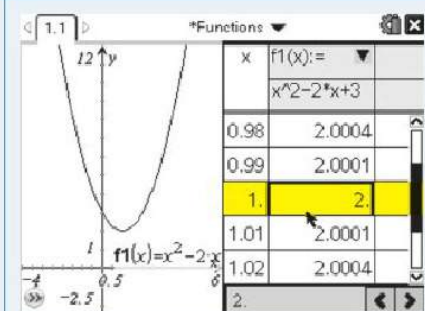
Table Step:

Independent:

Dependent:

OK **Cancel**

The table shows that the function has larger values at points around (1, 2). We can conclude that the point (1, 2) is a local minimum on the curve.



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Method 2: Using the minimum function

Press **menu** 6:Analyze Graph | 2:Minimum

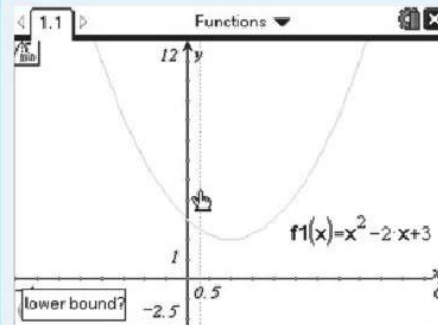
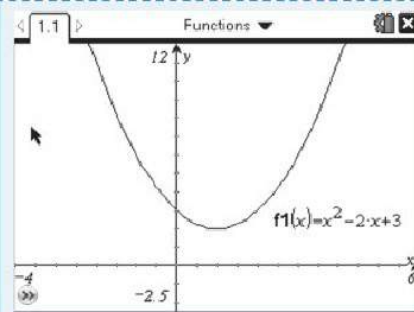
Press **enter**

To find the minimum you need to give the lower and upper bounds of a region that includes the minimum.

The GDC shows a line and asks you to set the lower bound.

Move the line using the touchpad and choose a position to the left of the minimum.

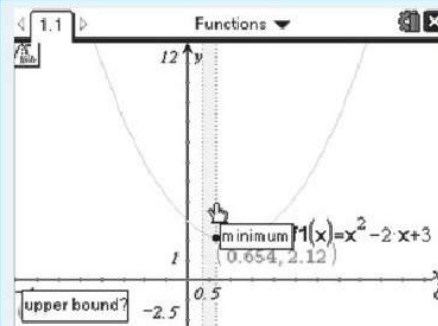
Click the touchpad.



The GDC shows another line and asks you to set the upper bound.

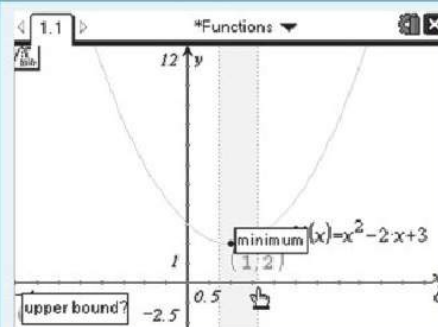
Use the touchpad to move the line so that the region between the upper and lower bounds contains the minimum.

Note: The minimum point in the region that you have defined is being shown. In this screenshot it is not the local minimum point. Make sure you move the line beyond the point you are looking for.

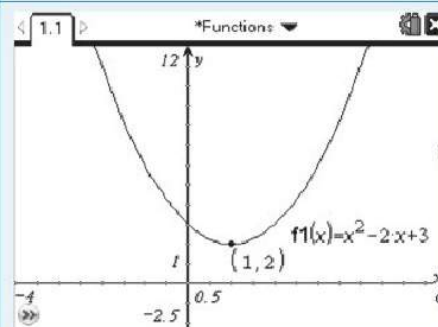


When the region contains the minimum, the GDC will display the word 'minimum' in a box and a point that lies between the lower and upper bounds. The point displayed is clearly between the upper and lower bounds.

Click the touchpad.



The calculator displays the minimum point on the curve at (1, 2).



Example 9

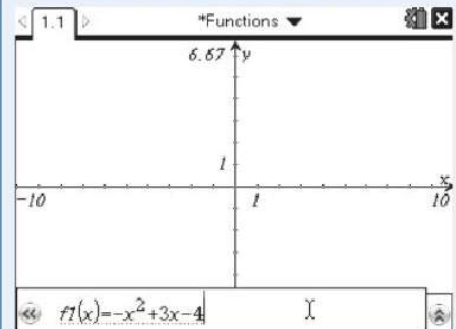
Find the maximum point on the graph of $y = -x^2 + 3x - 4$

First draw the graph of $y = -x^2 + 3x - 4$:

Open a new document and add a Graphs page.

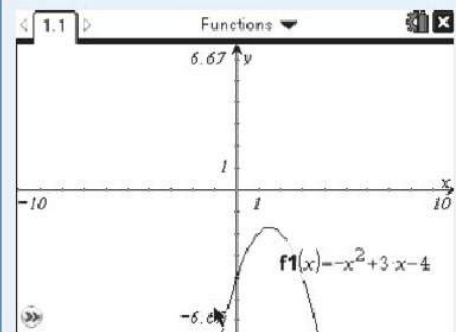
The entry line is displayed at the bottom of the work area. The default graph type is Function, so the form ' $f1(x)=$ ' is displayed.

The default axes are $-10 \leq x \leq 10$ and $-6.67 \leq y \leq 6.67$.



Type $-x^2 + 3x - 4$ and press **enter**.

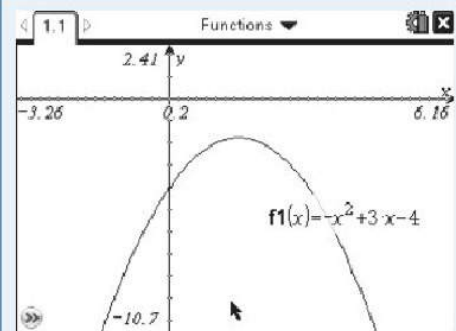
The GDC displays the curve with the default axes.



Pan the axes to get a better view of the curve.

Grab the x -axis and change it to make the quadratic curve fit the screen better.

For help with panning or changing axes, see your GDC manual.



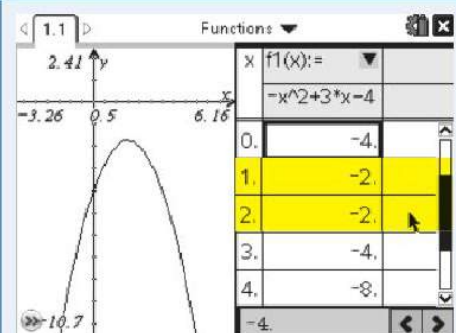
Method 1: Using a table

You can look at the graph **and** a table of the values by using a split screen.

Press **menu** 2:View | 9:Show Table

(or simply press **ctrl** **T**)

The maximum value shown in the table is -2 when $x = 1$ and $x = 2$.



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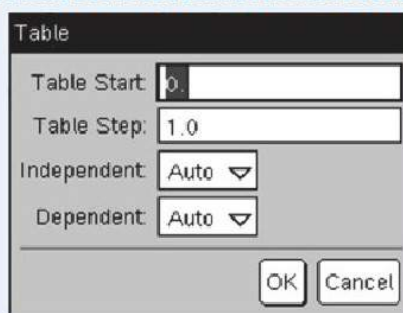
Look more closely at the values of the function between $x = 1$ and $x = 2$.

Change the settings in the table.

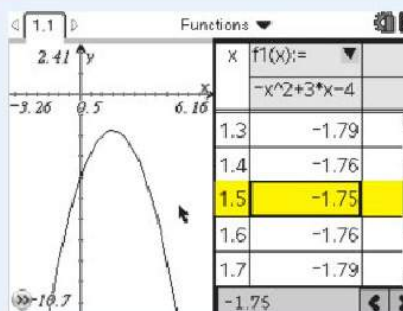
Choose any cell and press **menu** 5:Table | 5:Edit Table Settings...

Set Table Start to 1.0 and Table Step to 0.1.

Press **enter**



Scroll down the table and you can see that the function has its largest value at $(1.5, -1.75)$. We can conclude that the point $(1.5, -1.75)$ is a local maximum on the curve.



Method 2: Using the maximum function

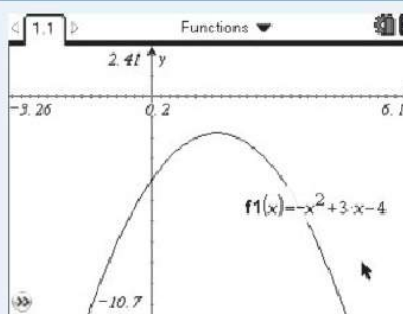
Press **menu** 6:Analyze Graph | 3:Maximum

Press **enter**

To find the maximum you need to give the lower and upper bounds of a region that includes the maximum. The GDC shows a line and asks you to set the lower bound.

Move the line using the touchpad and choose a position to the left of the maximum.

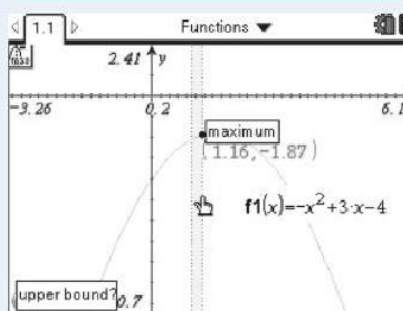
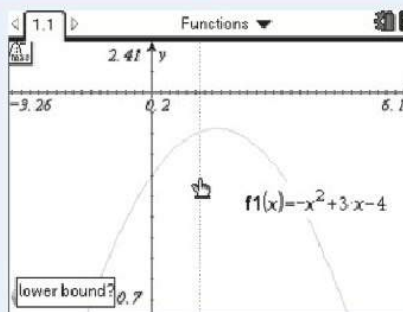
Click the touchpad.



The GDC shows another line and asks you to set the upper bound.

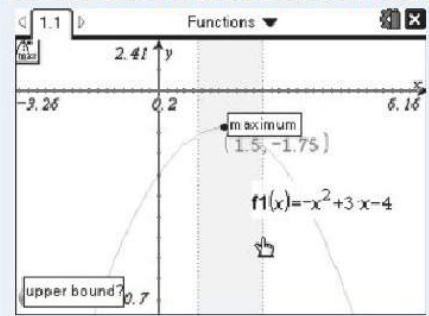
Use the touchpad to move the line so that the region between the upper and lower bounds contains the maximum.

Note: The maximum point in the region that you have defined is being shown. In this screenshot it is not the local maximum point. Make sure you move the line beyond the point you are looking for.

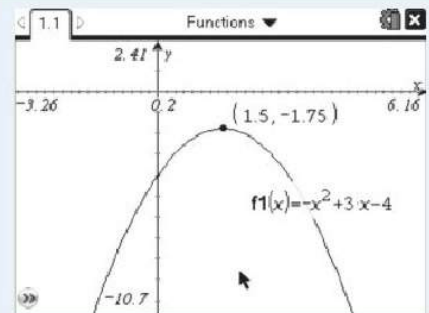


▶ Continued on next page

When the region contains the maximum, the GDC will display the word 'maximum' in a box and a point that lies between the lower and upper bounds. The point displayed is clearly between the upper and lower bounds. Click the touchpad.



The GDC displays the maximum point on the curve at (1.5, -1.75).



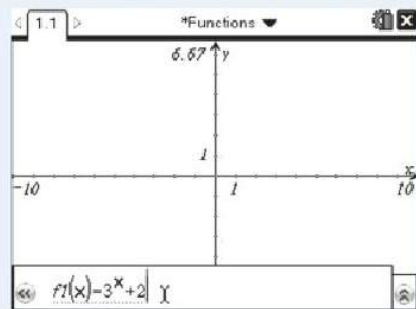
Exponential functions

1.9 Drawing an exponential graph

Example 10

Draw the graph of $y = 3^x + 2$

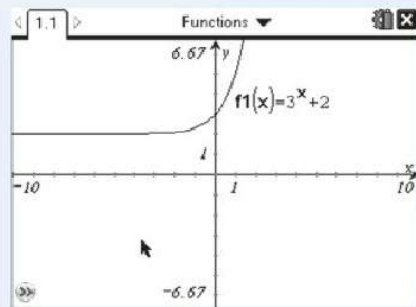
Open a new document and add a Graphs page. The entry line is displayed at the bottom of the work area. The default graph type is Function, so the form ' $f1(x) =$ ' is displayed. The default axes are $-10 \leq x \leq 10$ and $-6.67 \leq y \leq 6.67$.



Type $y = 3^x + 2$ and press **enter**.

(**Note:** Type **3** **^** **x** **▶** to enter 3^x . The **▶** returns you to the baseline from the exponent.)

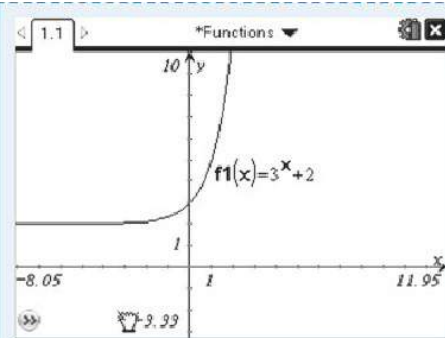
The GDC displays the curve with the default axes.



▶ Continued on next page

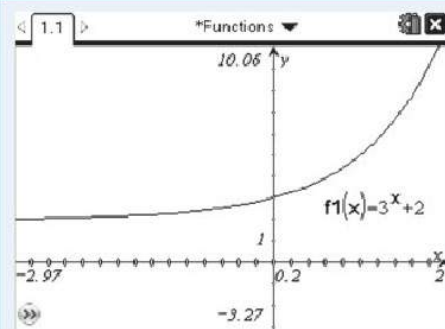
Pan the axes to get a better view of the curve.

For help with panning, see your GDC manual.



Grab the x -axis and change it to make the exponential curve fit the screen better.

For help with changing axes, see your GDC manual.

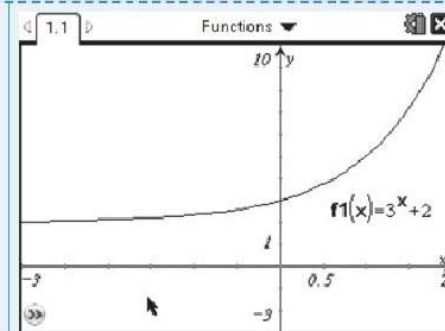


1.10 Finding a horizontal asymptote

Example 11

Find the horizontal asymptote to the graph of $y = 3^x + 2$

First draw the graph of $y = 3^x + 2$ (see Example 10).

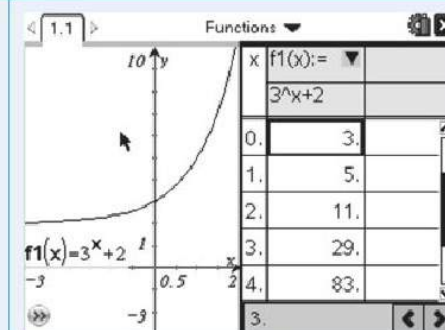


You can look at the graph **and** a table of the values by using a split screen.

Press **menu** 2:View | 9:Show Table

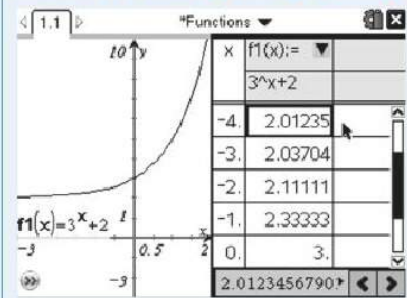
(or simply press **ctrl** **T**)

The values of the function are clearly decreasing as $x \rightarrow 0$.

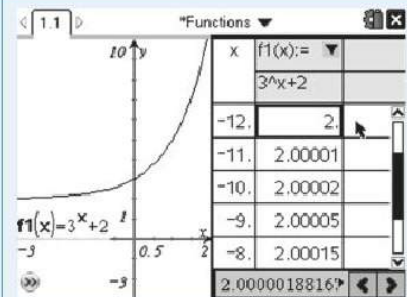


▶ Continued on next page

Press and hold \blacktriangle to scroll up the table.
The table shows that as the values of x get smaller, $f(x)$ approaches 2.



Eventually, the value of $f(x)$ reaches 2. On closer inspection, you can see, at the bottom of the screen, that the actual value of $f(x)$ is 2.0000018816...
We can say that $f(x) \rightarrow 2$ as $x \rightarrow -\infty$.
The line $x = 2$ is a horizontal asymptote to the curve $y = 3^x + 2$.



Logarithmic functions

1.11 Evaluating logarithms

Example 12

Evaluate $\log_{10} 3.95$, $\ln 10.2$ and $\log_5 2$.

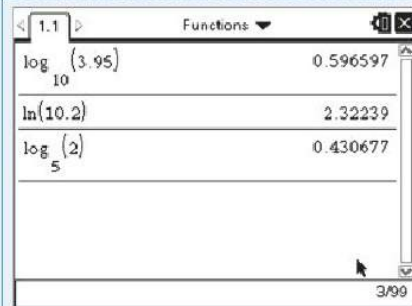
Open a new document and add a Calculator page.

Press ctrl log to open the log template.

Enter the base and the argument then press enter del

For natural logarithms it is possible to use the same method, with the base equal to e , but it is far less time consuming to press ctrl ln .

Note that the GDC will evaluate logarithms with any base without having to use the change of base formula.



1.12 Finding an inverse function

The inverse of a function can be found by interchanging the x and y values. Geometrically this can be done by reflecting points in the line $y = x$.

Example 13

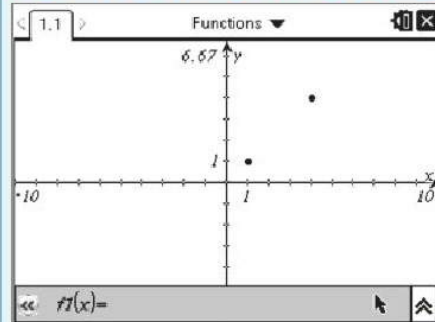
Show that the inverse of the function $y = 10^x$ is $y = \log_{10} x$ by reflecting $y = 10^x$ in the line $y = x$.

Open a new document and add a Graphs page.
First we will draw the line $y = x$. So that it can be recognised the axis of reflection, it has to be drawn and not plotted as a function.

Press **menu** 7: Points & Lines | 1: Point

Then type (1 enter 1 enter then (4 enter 4 enter esc

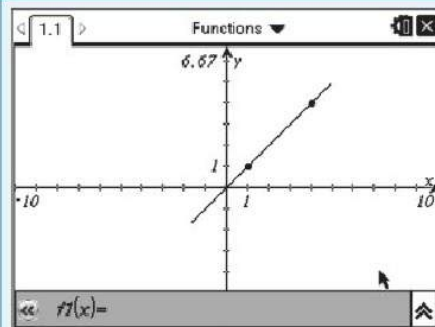
This will plot the points (1, 1) and (4, 4), which both lie on the line $y = x$



Press **menu** 7: Points & Lines | 4: Line

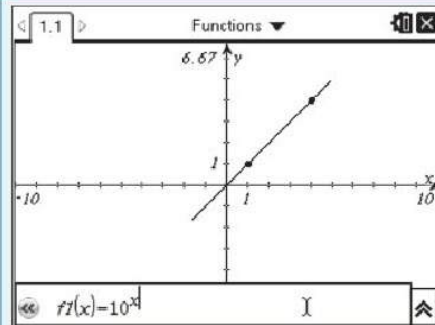
Select both the points you have plotted and draw a line through them.

Press **esc** to exit the drawing function.

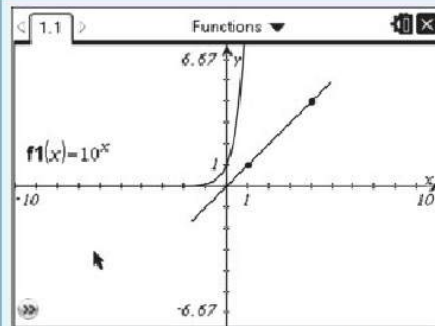


Click in the entry line at the bottom of the work area. The default graph type is Function, so the form " $f1(x)=$ " is displayed.

Type 10^x and press **enter**.

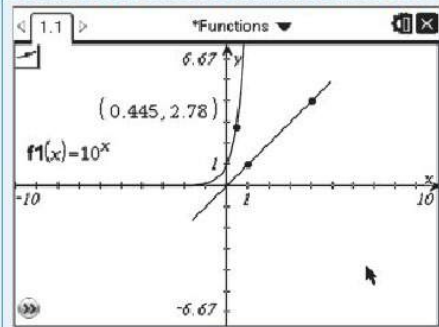


The calculator displays the function with the default axes, $-10 \leq x \leq 10$ and $-6.67 \leq y \leq 6.67$.

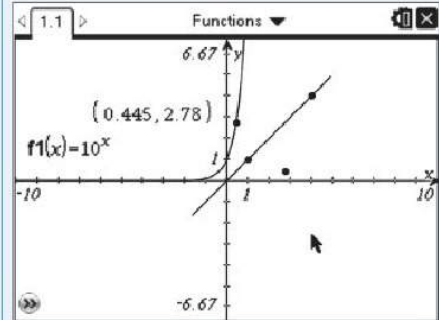


▶ Continued on next page

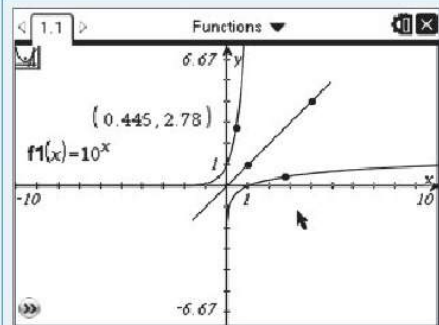
Press **menu** 7: Points & Lines | 2: Point On
 Select the curve with the touchpad (you will see that it is highlighted when it is selected).
 You can place a point anywhere on the curve.



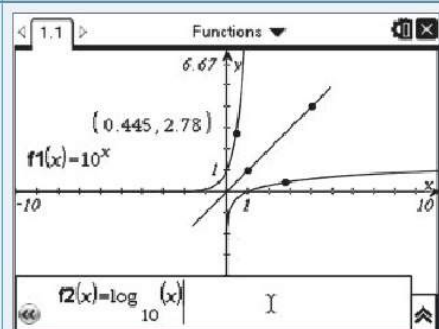
Press **menu** B: Transformation | 2: Reflection
 Use the touchpad to select the point that you just placed on the curve and then the line $y = x$.
 Press **esc** when you have finished. You should see the reflected image of the point in the line $y = x$.



Press **menu** A: Construction | 6: Locus
 Use the touchpad to select each of the points. The calculator will display the locus of the reflection as the point moves along the curve.



Click in the entry line at the bottom of the work area.
 “ $f2(x)=$ ” is displayed.
 Type $\log_{10}(x)$ and press **enter**.
 The reflected curve and the logarithmic function coincide, showing that $y = \log_{10}x$ is inverse of the function $y = 10^x$.



1.13 Drawing a logarithmic graph

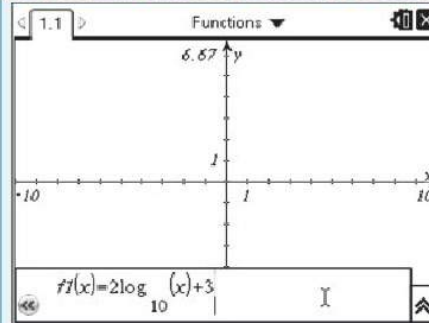
Example 14

Draw the graph of $y = 2\log_{10}x + 3$.

Open a new document and add a Graphs page.

The entry line is displayed at the bottom of the work area.
The default graph type is Function, so the form “ $f1(x)=$ ” is displayed.

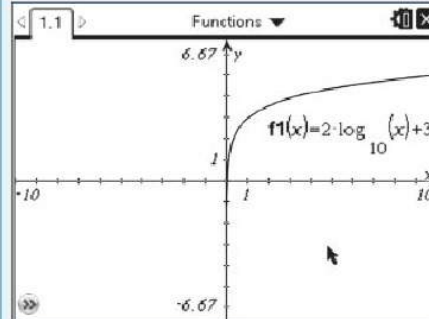
The default axes are $-10 \leq x \leq 10$ and $-6.67 \leq y \leq 6.67$.



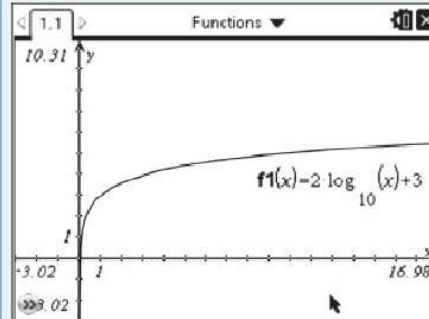
Type $2\log_{10}(x) + 3$ and press **enter**.

(Note: Type **2** **ctrl** **log** and enter 10 as the base of the logarithm. Enter x in the argument section of the template, use the **▶** to move beyond the brackets to enter $+3$)

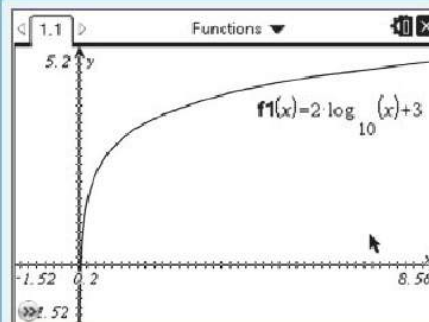
The calculator displays the curve with the default axes.



Pan the axes to get a better view of the curve.



Grab the x -axis and change it to make the logarithmic curve fit the screen better.





Trigonometric functions

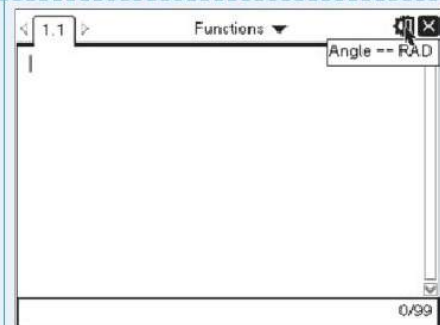
1.14 Degrees and radians

Work in trigonometry will be carried out either in degrees or radians. It is important, therefore, to be able to check which mode the calculator is in and to be able to switch back and forth. On the TI-Nspire, there are three separate settings to make: general, graphing and geometry. The defaults for general and graphing are radians and for geometry the default is degrees. Geometry is only used for drawing plane geometrical figures. Normally the two important settings are general and graphing. General refers to the angle used in calculations and graphing is for drawing trigonometric graphs.

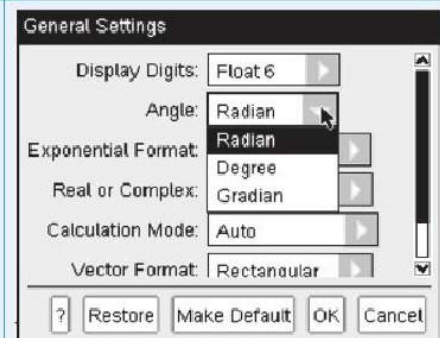
Example 15


Change angle settings from radians to degrees and from degrees to radians.

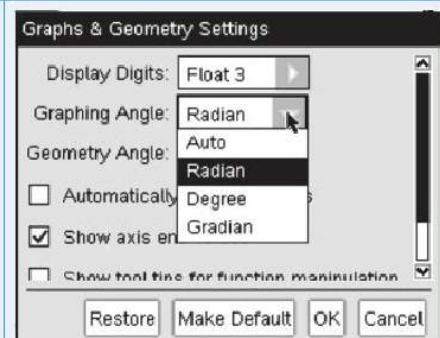
Open a new document and add a Calculator page.
Move the cursor to the  symbol at the top right hand side of the screen. It will display the *general* angle mode – either radians or degrees.
Click in the  symbol and choose 2:Settings | 1:General.



In the dialogue box, select either degrees or radians and then click on OK.



To change the setting for graphing, click in the  symbol and choose 2:Settings | 2:Graphs & Geometry.
In the dialogue box, select either degrees or radians for the Graphing Angle and then click on OK.



1.15 Drawing trigonometric graphs

Example 16

Draw the graph of $y = 2\sin\left(x + \frac{\pi}{6}\right) + 1$.

Open a new document and add a Graphs page.
 Press **menu** 4:Window / Zoom | 8:Zoom - Trig
 The entry line is displayed at the bottom of the work area.
 The default graph type is Function, so the form " $f1(x)=$ " is displayed.
 The default axes are $-6.28 \leq x \leq 6.28$ and $-4.19 \leq y \leq 4.19$.
 These are the basic axes for graphing trigonometric graphs with x between -2π and 2π . If the calculator is in degree mode, the x -axis will be between -360 and 360 .

Type $y = 2\sin\left(x + \frac{\pi}{6}\right) + 1$ and press **enter**.

To enter sin, press **trig** and choose sin from the dialogue box.

sin	cos	tan	csc	sec	cot
sin ⁻¹	cos ⁻¹	tan ⁻¹	csc ⁻¹	sec ⁻¹	cot ⁻¹

To enter π , press **π** and choose π from the dialogue box.

π	i	∞	e
θ	r	g	

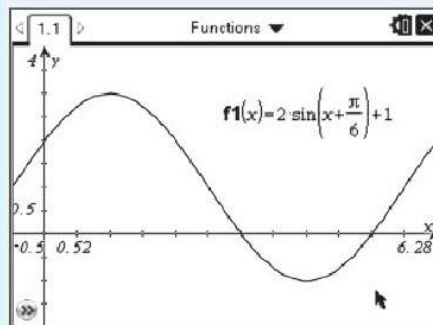
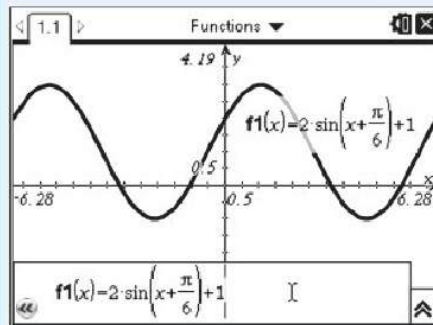
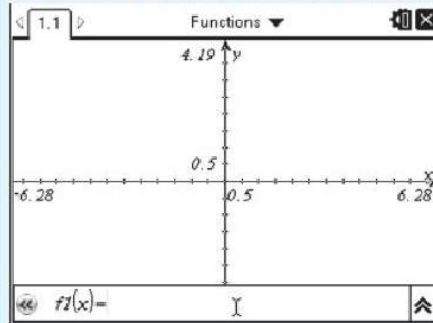
Pan the axes to get a better view of the curve and grab them to change the view.

It is also useful to change the x -axis scale to a multiple of π , such as $\frac{\pi}{6}$ as this will often show the positions of intercepts and turning points more clearly.

Change the scale by pressing **menu** 4:Window / Zoom | 1:Window Settings

XScale:

Type $\pi/6$ in the dialogue box for XScale.



More complicated functions

1.16 Solving a combined quadratic and exponential equation

Example 17

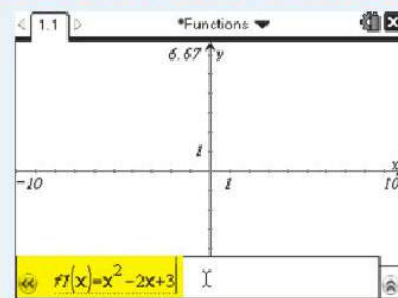
Solve the equation $x^2 - 2x + 3 = 3 \cdot 2^{-x} + 4$

To solve the equation, find the point of intersection of the quadratic function $f1(x) = x^2 - 2x + 3$ with the exponential function $f2(x) = 3 \cdot 2^{-x} + 4$.

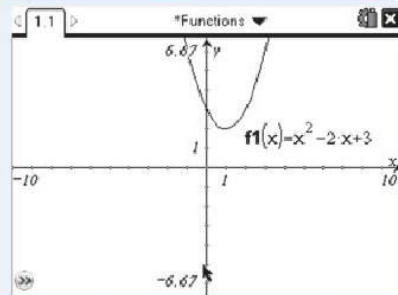
Follow the same GDC procedure when solving simultaneous equations graphically or solving a combined quadratic and exponential equation. See Examples 4 and 17.

To draw the graphs $f1(x) = x^2 - 2x + 3$ and $f2(x) = 3 \cdot 2^{-x} + 4$:

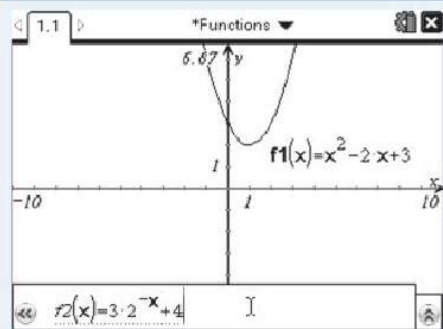
Open a new document and add a Graphs page.
The entry line is displayed at the bottom of the work area.
The default graph type is Function, so the form ' $f1(x)=$ ' is displayed.
The default axes are $-10 \leq x \leq 10$ and $-6.67 \leq y \leq 6.67$.



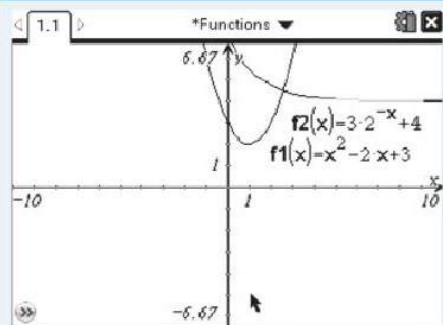
Type $x^2 - 2x + 3$ and press **enter**.
The GDC displays the first curve:
 $f1(x) = x^2 - 2x + 3$



Use the touchpad to click on the arrows in the bottom left-hand corner of the screen.
This will open the entry line again. This time ' $f2(x)=$ ' is displayed.
Type $3 \cdot 2^{-x} + 4$ and press **enter**.



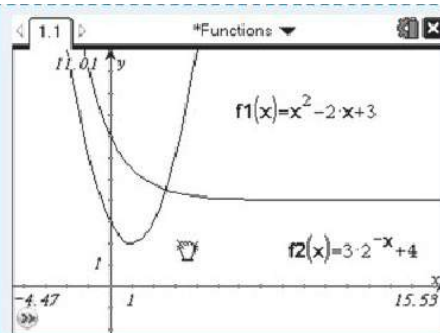
The GDC displays both curves:
 $f1(x) = x^2 - 2x + 3$
 $f2(x) = 3 \cdot 2^{-x} + 4$



▶ Continued on next page

Pan the axes to get a better view of the curves.

For help with panning, see your GDC manual.



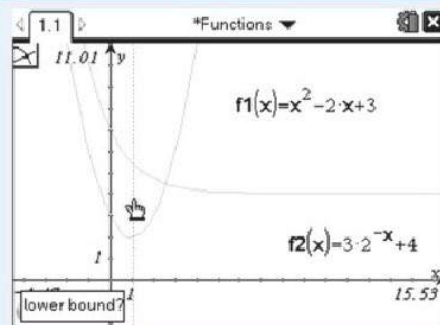
Press **menu** 6:Analyze Graph | 4:Intersection Point(s)

Press **enter**

To find the intersection you need to give the lower and upper bounds of a region that includes the intersection. The GDC shows a line and asks you to set the lower bound.

Move the line using the touchpad and choose a position to the left of the intersection.

Click the touchpad.

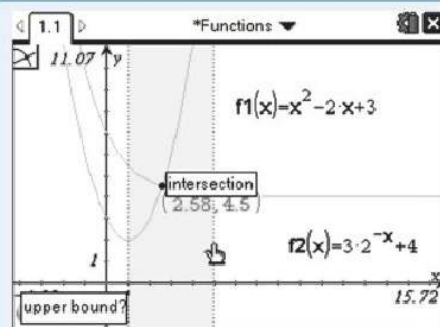


The GDC shows another line and asks you to set the upper bound.

Use the touchpad to move the line so that the region between the upper and lower bounds contains the intersection.

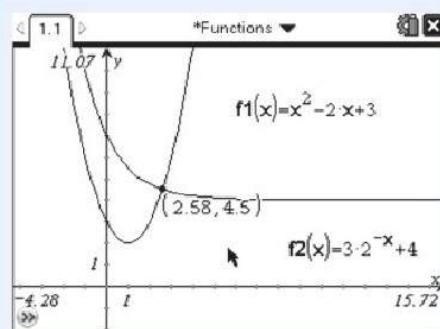
When the region contains the intersection, the calculator will display the word 'intersection' in a box.

Click the touchpad.



The GDC displays the intersection of the two curves at the point (2.58, 4.5).

The solution is $x = 2.58$.



Modeling

1.17 Using sinusoidal regression

Note: the notation $\sin^2 x$, $\cos^2 x$, $\tan^2 x$, ... is a mathematical convention that has little algebraic meaning. To enter these functions on the GDC, you *should* enter $(\sin(x))^2$, etc. However, the calculator will conveniently interpret $\sin(x)^2$ and translate it as $(\sin(x))^2$.

Example 18

It is known that the following data can be modeled using a sine curve.

x	0	1	2	3	4	5	6	7
y	6.9	9.4	7.9	6.7	9.2	8.3	6.5	8.9

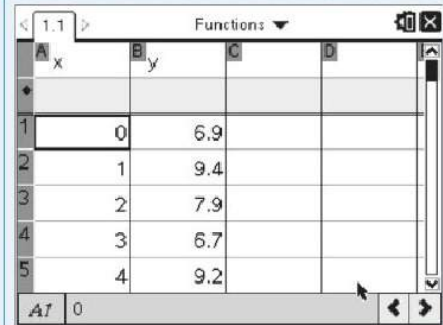
Use sine regression to find a function to model this data.

Open a new document and add a Lists & Spreadsheet page.

Type 'x' in the first cell and 'y' in the cell to its right.

Type the numbers from the x-list in the first column and those from the y-list in the second.

Use the \blacktriangledown \blacktriangle \blacktriangleleft \blacktriangleright keys to navigate around the spreadsheet.



Press 2nd On and add a new graphs page to your document.

Press 2nd 3 :Graph Type | 4:Scatter Plot

Press enter

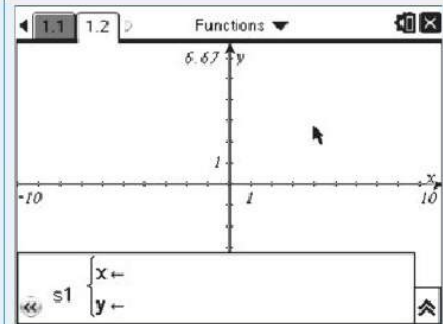
The entry line is displayed at the bottom of the work area.

Scatter plot type is displayed.

Enter the names of the lists, x and y, into the scatter plot function

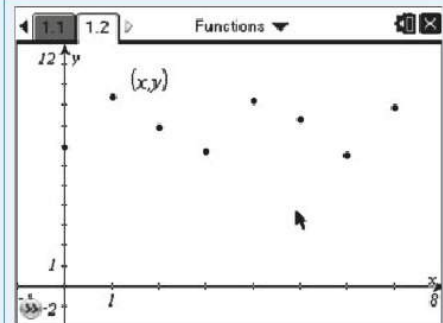
Use the tab key to move from x to y.

Press enter del



Adjust your window settings to show your data and the x- and y-axes.

You now have a scatter plot of x against y.



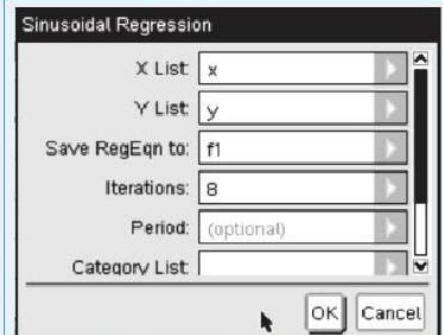
Press ctrl left to return to the Lists & Spreadsheet page.

Select an empty cell and press 2nd 4 :Statistics | Stat Calculations | C:Sinusoidal Regression...

Press enter

From the drop down menus choose 'x' for X List and 'y' for Y List. You should press tab to move between the fields.

Press enter



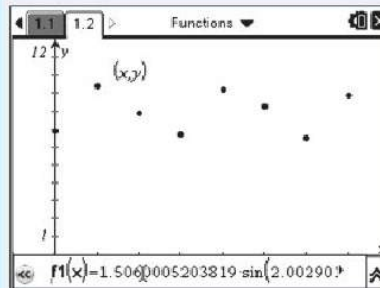
▶ Continued on next page

On screen, you will see the result of the sinusoidal regression in lists next to the lists for x and y . The equation is in the form $y = a\sin(bx + c) + d$ and you will see the values of a , b , c and d displayed separately. The equation of the sinusoidal regression line is $y = 1.51\sin(2.00x - 0.80) + 7.99$

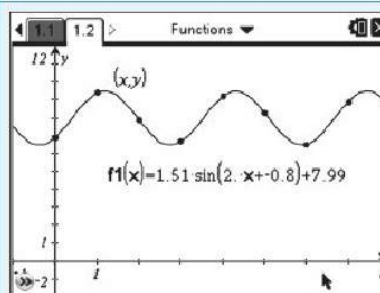
	A	B	C	D
	x	y		
				=SinReg(x)
1	0	6.9	Title	Sinoid...
2	1	9.4	RegEqn	a*sin(b*x...
3	2	7.9	a	1.506
4	3	6.7	b	2.0029
5	4	9.2	c	-0.799874

D1 = "Sinusoidal Regression"

Press **ctrl** to return to the Graphs page. Using the touchpad, click on **▶** to open the entry line at the bottom of the work area. You will see that the equation of the regression line has been pasted into $f1(x)$. Press **enter**



The regression line is now shown on the graph.



1.18 Using transformations to model a quadratic function

Example 19

This data is approximately connected by a quadratic function.

x	-2	-1	0	1	2	3	4
y	9.1	0.2	-4.8	-5.9	-3.1	4.0	15.0

Find a function that fits the data.

Open a new document and add a Lists & Spreadsheet page. Enter the data in two lists: Type 'x' in the first cell and 'y' in the cell to its right. Enter the x -values in the first column and the y -values in the second. Remember to use **(-)** to enter a negative number. Use the **▼ ▲ ◀ ▶** keys to navigate around the spreadsheet.

	A	B	C	D
	x	y		
1	-2	9.1		
2	-1	0.2		
3	0	-4.8		
4	1	-5.9		
5	2	-3.1		

A7 -2

You can also model a linear function by finding the equation of the least squares regression line (see section 5.15).

Transform a basic quadratic curve to find an equation to fit some quadratic data.

▶ Continued on next page

Add a Graphs page to your document.
 Press **menu** 3:Graph Type | 4:Scatter Plot

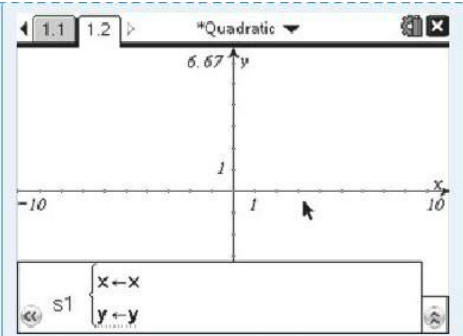
Press **enter**

The entry line is displayed at the bottom of the work area.
 Scatter plot type is displayed.

Enter the names of the lists, x and y , into the scatter plot function.

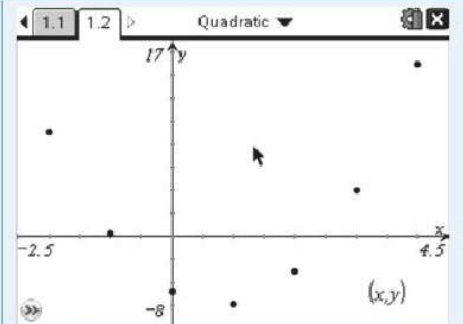
Use the **tab** key to move from x to y .

Press **enter**



Press **menu** A:Zoom – Fit from the Window/Zoom menu
 This is a quick way to choose an appropriate scale to show all the points.

You should recognize that the points are in the shape of a quadratic function.



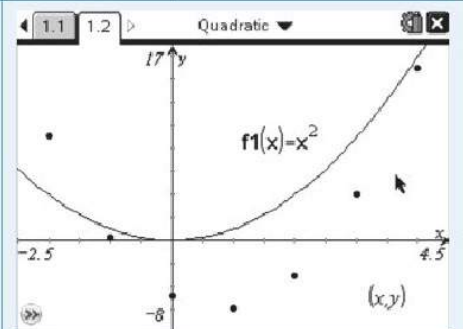
The next step is to enter a basic quadratic function,
 $y = x^2$, and manipulate it to fit the points.

Press **menu** 3:Graph Type | 1:Function

Press **enter**

This changes the graph type from scatter plot to function.
 Type x^2 in as function $f1(x)$.

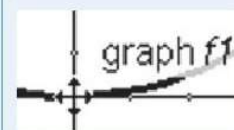
It is clear that the curve does not fit any of the points, but it is the right general shape to do so.



Use the touchpad to move the cursor so it approaches the curve. You will see one of two icons.

The first will allow you to drag the quadratic function around the screen by its vertex.

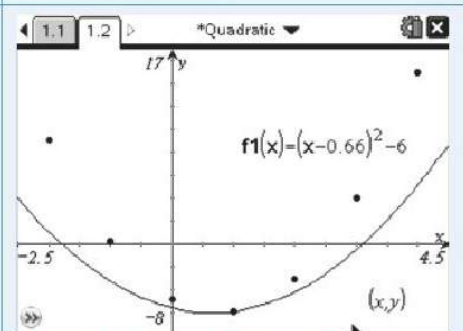
The second allows you to stretch the function either vertically or horizontally.



or



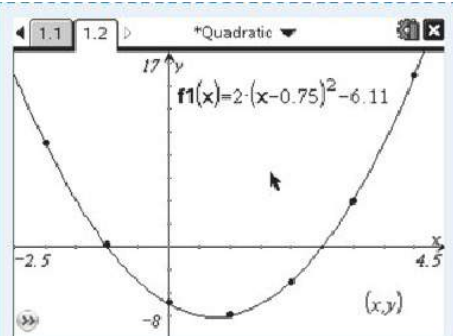
Use $\leftarrow \rightarrow$ to position the vertex where you think it ought to be according to the data points.



▶ Continued on next page

Use \times to adjust the stretch of the curve.
 Make some final fine adjustments using both the tools until you have a good fit to the data points.
 The equation of the function that fits the data is:

$$f(x) = 2(x - 0.75)^2 - 6.11$$



1.19 Using sliders to model an exponential function

Example 20

In general, an exponential function has the form $y = ka^x + c$.
 For this data, it is known that the value of a is 1.5, so $y = k(1.5)^x + c$.

x	-3	-2	-1	0	1	2	3	4	5	6	7	8
y	3.1	3.2	3.3	3.5	3.8	4.1	4.7	5.5	6.8	8.7	11.5	15.8

Find the values of the constants k and c .

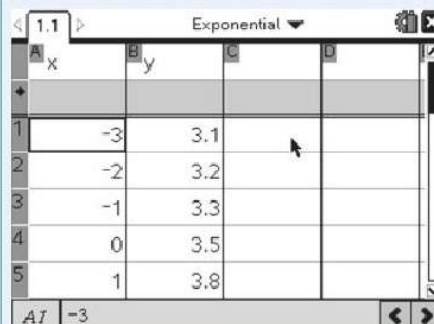
Open a new document and add a Lists & Spreadsheet page.

Enter the data in two lists:

Type 'x' in the first cell and 'y' in the cell to its right.

Enter the x -values in the first column and the y -values in the second. Remember to use $(-)$ to enter a negative number.

Use the ∇ \blacktriangle \blacktriangleleft \blacktriangleright keys to navigate around the spreadsheet.



Add a Graphs page to your document.

Press menu 3:Graph Type | 4:Scatter Plot

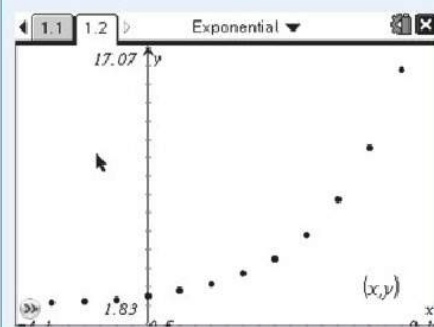
Press enter

The entry line is displayed at the bottom of the work area.
 Scatter plot type is displayed.

Enter the names of the lists, x and y , into the scatter plot function.

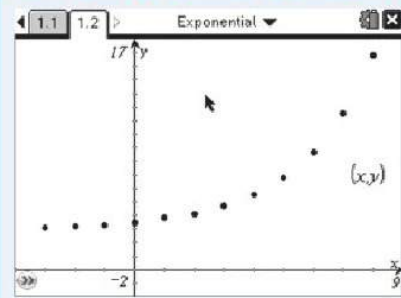
Use the tab key to move from x to y .

Press enter



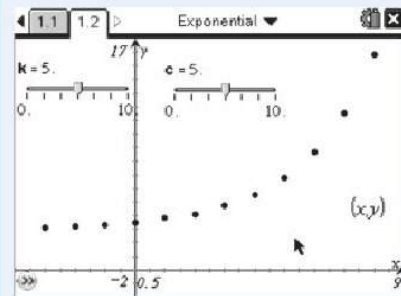
▶ Continued on next page

Adjust the window settings to fit the data and to display the axes clearly.

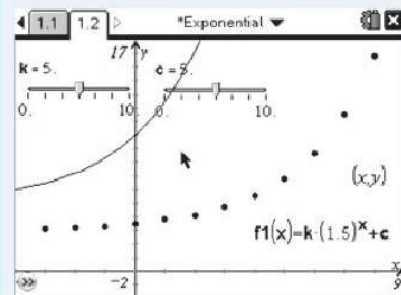


Press **menu** I:Actions | A:Insert Slider
Position the slider somewhere where it is not in the way and change the name of the constant to k .
Repeat and add a second slider for c .

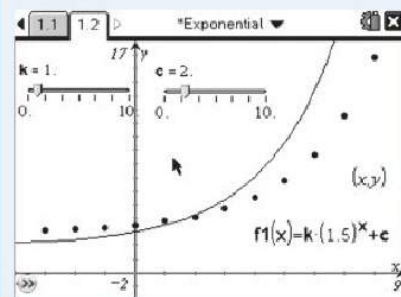
For help with sliders, see your GDC manual.



Press **menu** 3:Graph Type | 1:Function
Press **enter**
This changes the graph type from scatter plot to function.
Type $k \cdot (1.5)^x + c$ in as function $f1(x)$.



Try adjusting the sliders.
You can get the curve closer to the points but they are not sufficiently adjustable to get a good fit.



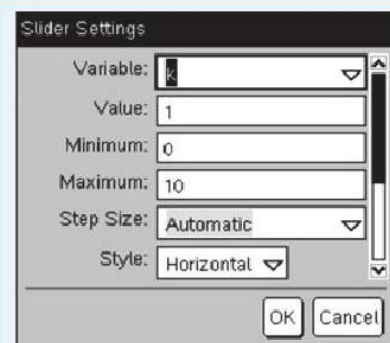
You can change the slider settings by selecting the slider, pressing **ctrl** **menu** and selecting 1:Settings.

Change the default values for k to:

- Minimum 0
- Maximum 2
- Step Size 0.1

Change the default values for c to:

- Minimum 0
- Maximum 4
- Step Size 0.1

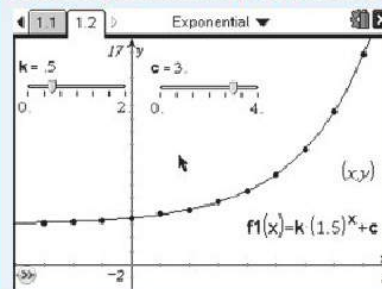


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You can now adjust the sliders to get a much better fit to the curve.

The screen shows the value of k is 0.5 and c is 3.

So the best fit for the equation of the function is approximately $y = 0.5(1.5)^x + 3$.



2 Differential calculus

Finding gradients, tangents and maximum and minimum points

2.1 Finding the gradient at a point

Example 21

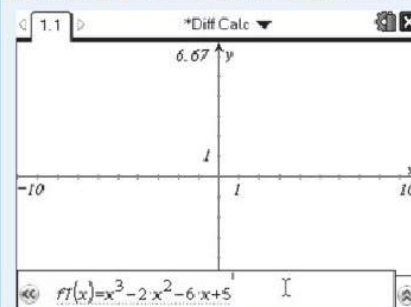
Find the gradient of the cubic function $y = x^3 - 2x^2 - 6x + 5$

Open a new document and add a Graphs page. The entry line is displayed at the bottom of the work area. The default graph type is Function, so the form ' $f1(x)=$ ' is displayed.

The default axes are $-10 \leq x \leq 10$ and $-6.67 \leq y \leq 6.67$.

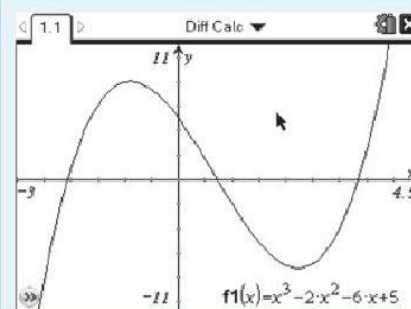
Type $x^3 - 2x^2 - 6x + 5$ and press **enter**.

(Note: Type **x** **^** **3** **▶** to enter x^3 . The **▶** returns you to the baseline from the exponent.)





Pan the axes to get a better view of the curve and then grab the x - and y -axes to fit the curve to the window.

For help with panning and changing axes, see your GDC manual.

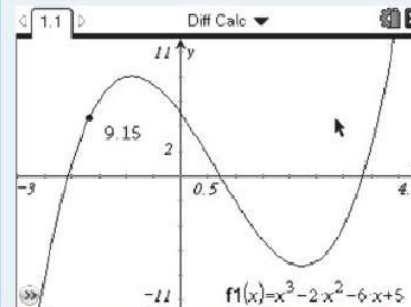


Press **menu** 6:Analyze Graph | 5: $\frac{dy}{dx}$

Press **enter**

Using the touchpad, move the  towards the curve. As it approaches the curve, it turns to  and displays the numerical value of the gradient.

Press **enter** to attach a point on the curve.



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