Answer Key

Q16/16/MATHSL11/SP1/ENG/TZ1

International Baccalaureate
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IB MATH SL
GRADE 11
Semester 2
Topic 3: Circular function
and Trigonometry

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Candidate name

FORM A

INSTRUCTION TO CANDIDATES

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are NOT permitted access to any calculator for this paper.
- At the end of the examination, indicate the number of sheets used in your cover.
- Unless otherwise stated in the question, all numerical answers must be given exactly (simplest fraction, simplest radical) or correct to three significant figures.

Objectives:

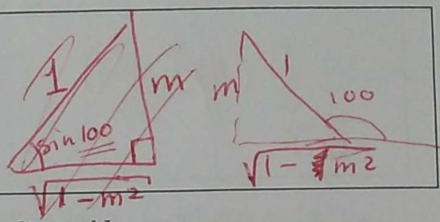
Students will be assessed on,

- The circle: radian measure of angles; length of an arc; area of a sector.
- Definition of cosine and sine terms of the unit circle.
- Relationship between trigonometric ratios
- Solution of triangles. The cosine rule. The sine rule, including the ambiguous case.

1a. Let $\sin 100° = m$. Find an expression for $\cos 100°$ in terms of m. *[3 marks]*

$\cos 100 = \sqrt{1-m^2}$

2^{nd} Q $= -\sqrt{1-m^2}$



1b. Let $\sin 100° = m$. Find an expression for $\tan 100°$ in terms of m. *[1 mark]*

$\tan 100 = \dfrac{m}{-\sqrt{1-m^2}} = \dfrac{\sin 100}{\cos 100}$

2a. The diagram shows two concentric circles with center O. *[2 marks]*

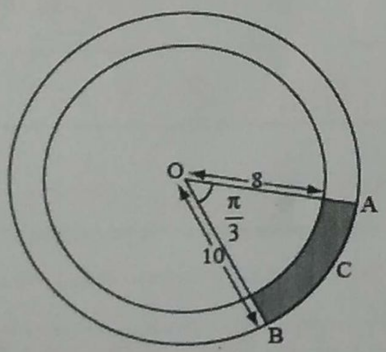


diagram not to scale

The radius of the smaller circle is 8 cm and the radius of the larger circle is 10 cm.

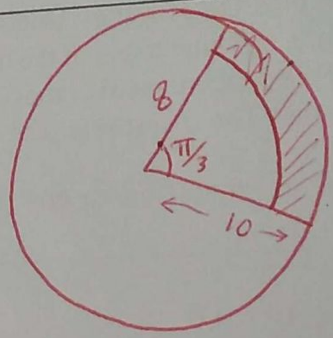
Points A, B and C are on the circumference of the larger circle such that $A\hat{O}B$ is $\frac{\pi}{3}$ radians.

Find the length of the arc ACB .

Arc $l = \theta r$

$= \dfrac{\pi}{3} \times 10 = \dfrac{10\pi}{3}$

2b. Find the area of the shaded region. *[4 marks]*



Shaded region.

$A_s = A_1 - A_2$

$= \frac{1}{2}\left(\frac{\pi}{3}\right)(10)^2 - \frac{1}{2}\left(\frac{\pi}{3}\right)(8)^2$

$= \frac{\pi}{6}(100-64)$ 36

$= 6\pi$

- Let $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$.

3a. Find $f\left(\frac{\pi}{2}\right)$. *[2 marks]*

$\cos 2\left(\frac{\pi}{2}\right) = \cos \pi = -1$

3b. Find $(g \circ f)\left(\frac{\pi}{2}\right)$ *[2 marks]*

$g \circ f\left(\frac{\pi}{2}\right) = 2\left(\cos 2\left(\frac{\pi}{2}\right)\right)^2 - 1$

$= 2(-1)^2 - 1 =$

$= +2 - 1 = 1$

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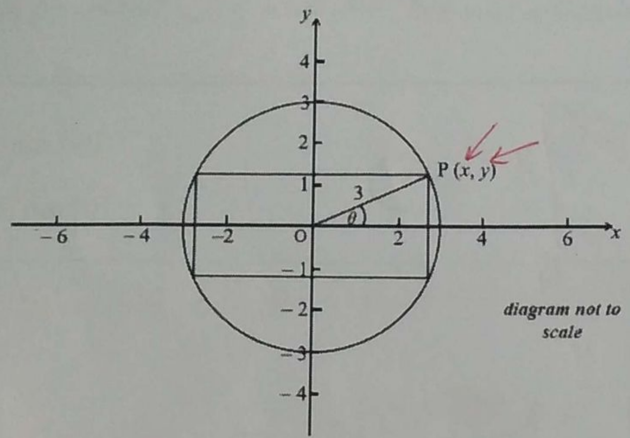
A rectangle is inscribed in a circle of radius 3 cm and centre O, as shown below.



The point P$(x\,,y)$ is a vertex of the rectangle and also lies on the circle. The angle between (OP) and the x-axis is θ radians, where $0 \leq \theta \leq \frac{\pi}{2}$.

4a. Write down an expression in terms of θ for *[3 marks]*

(i) x ;

(ii) y .

$\sin\theta = \frac{y}{3} \Rightarrow y = 3\sin\theta$

$\cos\theta = \frac{x}{3} \Rightarrow x = 3\sin\theta$

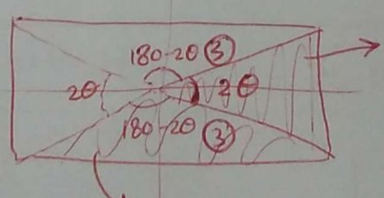
4b. Let the area of the rectangle be A. Show that $A = 18\sin 2\theta$ . . *[3 marks]*

$A = 18\sin\theta$?

$\therefore$ Area =

$4\left[\frac{1}{2}(3)(3)\sin 2\theta\right)$

$= 18\sin 2\theta$



Area $= \frac{1}{2}(3)(3)\sin 2\theta$

Area $= \frac{1}{2}(3)(3)\sin(180-2\theta)$

$= \frac{1}{2}(3)(3)\sin 2\theta$

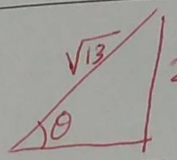
[3 marks]

5. Let $\sin\theta = \frac{2}{\sqrt{13}}$, where $\frac{\pi}{2} < \theta < \pi$.

Find $\tan\theta$.

1^{st} Q $\therefore \tan\theta = \frac{2}{3}$ (1)

2^{nd} Q or $\tan\theta = \frac{-2}{3}$ (1)



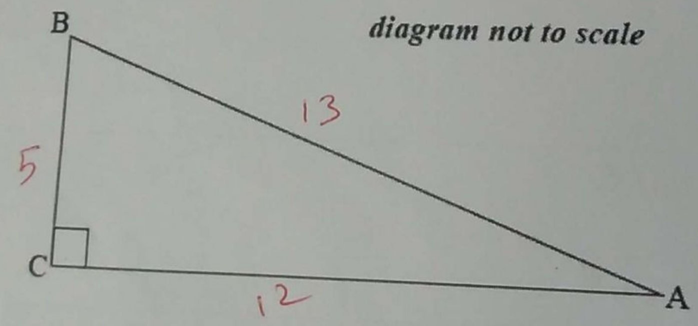
(1)

$\sqrt{(\sqrt{13})^2 - 4}$

$= \sqrt{13-4}$

$= \sqrt{9} = 3$

6. The following diagram shows a right-angled triangle, **ABC**, where $\sin A = \frac{5}{13}$.. [2 marks]



Show that $\cos A = \frac{12}{13}$.

Method (1)

$AC = \sqrt{13^2 - 5^2} = \sqrt{169-25} = \sqrt{144} = 12 \quad \therefore \cos A = \frac{12}{13}$

Method (2)

$\sin A^2 + \cos^2 A = 1$

$\left(\frac{5}{13}\right)^2 + \cos^2 A = 1$

$\cos A = \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{169-25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$

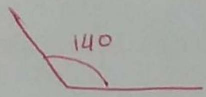
7a. *[2 marks]*

Let $p = \sin 40°$ and $q = \cos 110°$. Give your answers to the following in terms of p and/or q.

Write down an expression for

(i) $\sin 140°$;

(ii) $\cos 70°$.



- $\sin 40 = \sin(180-40) = \sin 140 = p$
- $\cos 70 = -\cos(180-\cancel{70}) = -\cos 110 = q$

$\therefore \cos 110 = -q$

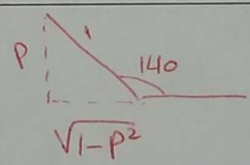
7b. *[3 marks]* Find an expression for $\cos 140°$.

$\cos 140 = \dfrac{\sqrt{1-p^2}}{1}$

$\downarrow$

Because it is in the 2^{nd} Q

$\cos 140 = -\sqrt{1-p^2}$



7c. *[1 mark]* Find an expression for $\tan 140°$.

$\tan 140 = \dfrac{\sin 140}{\cos 140} = \dfrac{p}{-\sqrt{1-p^2}}$

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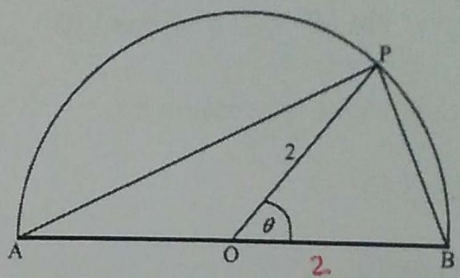
8a. *[2 marks]*

The following diagram shows a semicircle center O, diameter [AB], with radius 2.



Let P be a point on the circumference, with $\mathbf{P\hat{O}B} = \theta$ radians.

Find the area of the triangle OPB, in terms of θ.

$\text{Area} = \frac{1}{2}(2)(2)\sin\theta$

$= 2\sin\theta$

8b. *[3 marks]* Explain why the area of triangle OPA is the same as the area triangle OPB.

$\text{Area} = \frac{1}{2}(2)(2)\sin(180-\theta)$

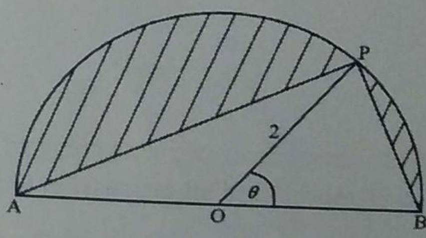
$= \frac{1}{2}(4)\sin\theta$

$= 2\sin\theta$

$\boxed{\sin\theta = \sin(180-\theta)}$

8c. *[3 marks]*

Let S be the total area of the two segments shaded in the diagram below.



Show that $S = 2(\pi - 2\sin\theta)$.

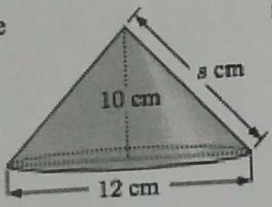
$\text{Area}_{\frac{1}{2}\text{circle}} = \frac{1}{2}\pi r^2 = 2\pi$

$\text{Area of}_{\Delta} = 2\sin\theta$

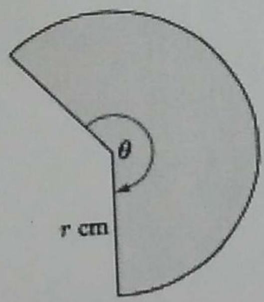
$\therefore S = 2\pi - 2\sin\theta - 2\sin\theta$

$= 2(\pi - 2\sin\theta)$

The cone



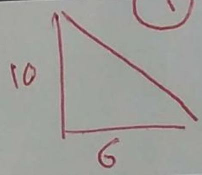
is made from this sector:



Keep all the answers as a radical or in terms of π

9a. the slant length s cm — 2 marks

$\sqrt{100+36} = \sqrt{136}$ 1



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9b. the arc length of the sector — 3 marks

~~A~~ Arc length = Parameter of the base of the Cone $= 2\pi r$ 1

$= 2\pi(6) = 12\pi$

1 1

9c. the value of r — 1 mark

$r = S = \sqrt{136}$ 1

9d. the sector angle θ in radians. — 2 marks

Arc length $= \theta r$

$12\pi = \theta\sqrt{136} \Rightarrow \theta = \frac{12\pi}{\sqrt{136}} = \frac{6\pi}{\sqrt{34}}$