O16/16/MATHSL11/SP1/ENG/TZ1

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IB MATH SL
GRADE 11

Semester 2
Topic 3: Circular function and Trigonometry
Test 2, March 21/2017

Candidate name

Answer Key

Raafe

41

FORM A

THE UNIVERSAL AMERICAN SCHOOL
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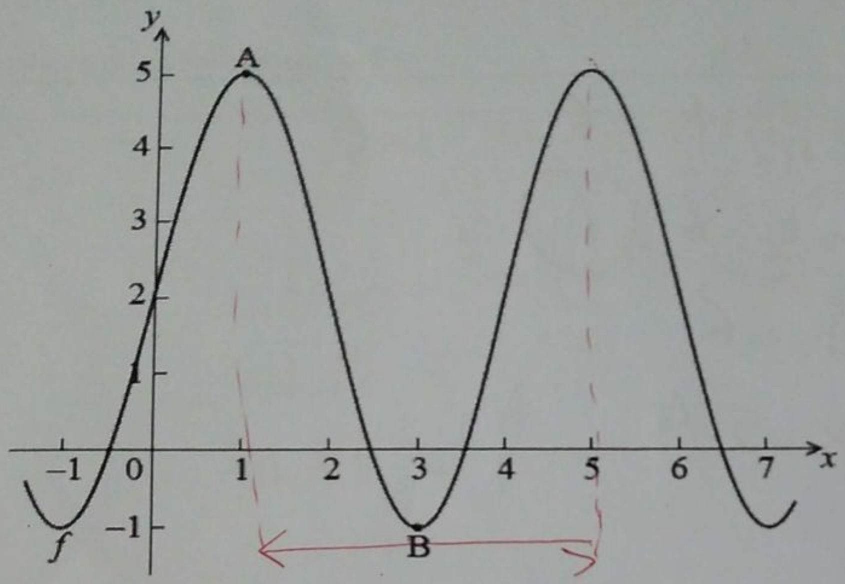
INSTRUCTION TO CANDIDATES

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are NOT permitted access to any calculator for this paper.
- At the end of the examination, indicate the number of sheet used in your cover.
- Unless otherwise stated in the question, all numerical answers must be given exactly (simplest fraction, simplest radical) or correct to three significant figures.

1. *[6 marks]* The diagram below shows part of the graph of a function f.



The graph has a maximum at A$(1, 5)$ and a minimum at B$(3, -1)$.

The function f can be written in the form $f(x) = p\sin(qx) + r$. Find the value of

(a) p

(b) q

(c) r.

M1 A1 A1

$p = \frac{max - min}{2} = \frac{5 - (-1)}{2} = 3$ N2

$q = \frac{2\pi}{period}$ period $= 4$ N2

$= \frac{2\pi}{4} = \frac{\pi}{2}$

$r = \frac{max + min}{2} = \frac{5 + (-1)}{2} = 2$ N2

2a. *[3 marks]* Let $f(x) = \sin\left(x + \frac{\pi}{4}\right) + k$. The graph of f passes through the point $\left(\frac{\pi}{4}, 6\right)$.

x y

Find the value of k.

M1

$\sin\left(\frac{\pi}{4} + \frac{\pi}{4}\right) + K = 6$

$\sin\frac{\pi}{2} + K = 6$ A1

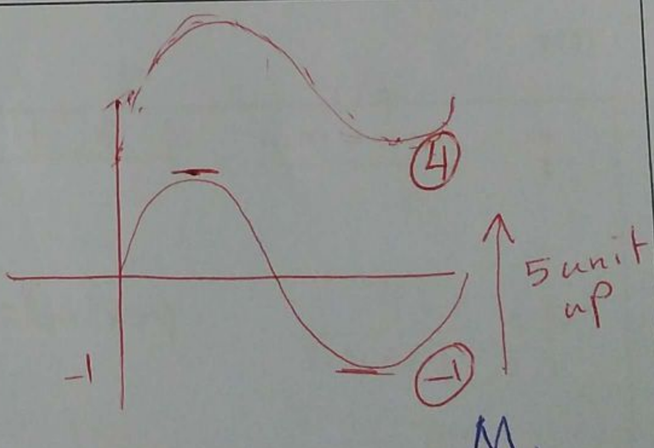
$1 + K = 6$

$K = 5$ A1 N2

b. *[2 marks]* Find the minimum value of $f(x)$.

$f(x) = \sin\left(x + \frac{\pi}{4}\right) + 5$

$-1 + 5 = \boxed{4}$ A1 N2



M1

2c. *[2 marks]* Let $g(x) = \sin x$. The graph of g is translated to the graph of f by the vector $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of p and of q.

$g(x) = \sin x$

$\downarrow$

$f(x) = \sin\left(x + \frac{\pi}{4}\right) + 5$

$\therefore$ shift $\frac{\pi}{4}$ left

shift 5 up

$\therefore p = -\frac{\pi}{4}$ 1

$q = 5$ 1

3a. *[1 mark]* Let $f(x) = 3\sin(\pi x)$. Write down the amplitude of f.

~~f(x)~~ The amplitude $= 3$ 1

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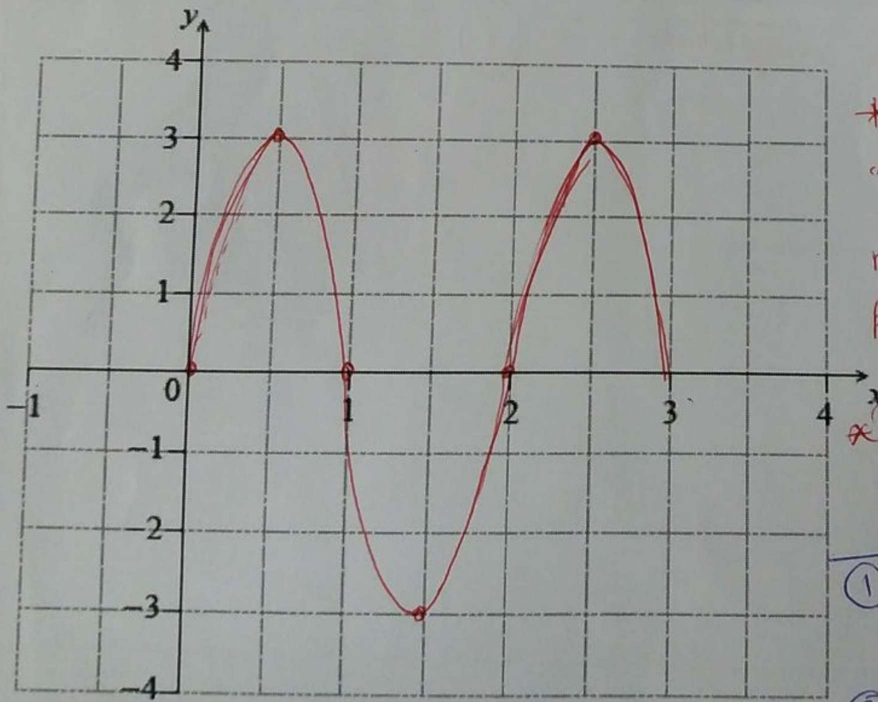
3b. *[2 marks]* Find the period of f.

The period =

$B = \frac{2\pi}{\text{period}} = \pi$ M

period $= \frac{2\pi}{B} = \frac{2\cancel{\pi}}{\cancel{\pi}} = 2$ A N2

3c. *[4 marks]* On the following grid, sketch the graph of $y = f(x)$, for $0 \le x \le 3$.



* period = 2

"every 2 we repeat the ~~f~~ function"

* The amplitude = 3

① Sin curve. (0,0) & correct period

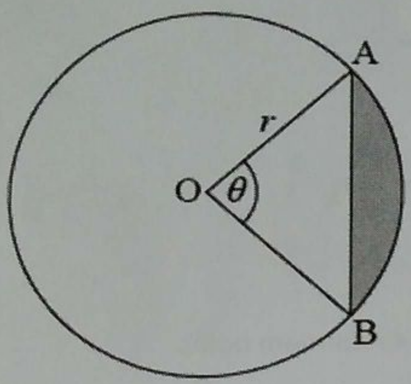
② x-int.

③ Max & Min.

④ Domain

4a. *[3 marks]*

A circle centre O and radius r is shown below. The chord [AB] divides the area of the circle into two parts. Angle AOB is θ.



Find an expression for the area of the shaded region in terms of θ and r.

$$A_{shaded} = A_{sector} - A_{triangle}$$

$$= \frac{1}{2}\theta r^2 - \frac{1}{2} r.r \sin\theta$$

$$= \frac{1}{2}\theta r^2 - \frac{1}{2} r^2 \sin\theta$$

$$= \frac{1}{2} r^2(\theta - \sin\theta)$$

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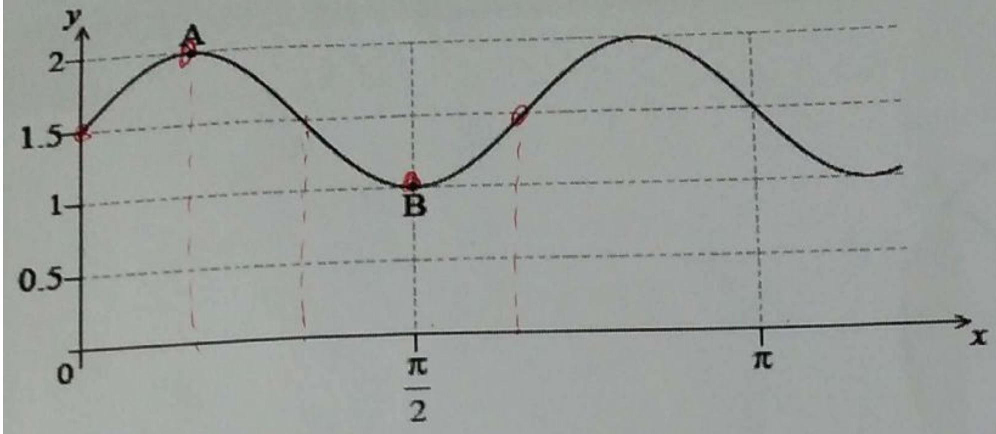
The following diagram shows part of the graph of $y = p\sin(qx) + r$.



The point $A\left(\frac{\pi}{6}, 2\right)$ is a maximum point and the point $B\left(\frac{\pi}{2}, 1\right)$ is a minimum point.

Find the value of

5a. *[2 marks]* p;

amplitude $= p = \dfrac{\text{max} - \text{min}}{2} = \dfrac{2-1}{2} = \dfrac{1}{2}$ A_1 N_2

M_1

5b. *[2 marks]* r;

$r = \dfrac{\text{max} + \text{min}}{2} = \dfrac{2+1}{2} = 1.5$ A_1 N_2

M_1

5c. *[2 marks] q.*

$q = \frac{2\pi}{\text{period}} =$

$= \frac{\cancel{2\pi}}{\cancel{2\pi}/3} = 3$

$\boxed{q = 3}$ A1 N2

$\frac{1}{2}$ period $= \frac{\pi}{2} - \frac{\pi}{6}$

$\frac{1}{2}$ period $= \frac{3\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$

period $= \frac{\pi}{3} \times 2$

$= \frac{2\pi}{3}$

A1

6a. [3 marks] Factorize:

$$2\sin^2 x + 7\sin x \cos x + 3\cos^2 x = 0$$

$\sin x = a$

$\cos x = b$

$2a^2 + 7ab + 3b^2 = 0$

1 $\underbrace{2a^2 + ab} + 6ab + 3b^2 = 0$

$1 \times 6 = 6$

$1 + 6 = 7$

1 $a(2a + b) + 3b(2a + b) = 0$

$(2a + b)(a + 3b) = 0$

1 $(2\sin x + \cos x)(\sin x + 3\cos x) = 0$

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$1-\frac{\cos^2 x}{1+\sin x}\times\frac{(1-\sin x)}{(1-\sin x)}$

$1-\frac{\cos^2 x(1-\sin x)}{\cancel{1-\sin^2 x}}=1-1+\sin x$
$=\sin x$

$1-\frac{1-\sin^2 x}{1+\sin x}=1-\frac{(1-\sin x)(\cancel{1+\sin x})}{\cancel{1+\sin x}}$
$=1-(1-\sin x)=1-1+\sin x$
$=\sin x$

6b. [6 marks] Simplify:

or

i) $1-\frac{\cos^2 x}{1+\sin x}$

or

(1) $\frac{1+\sin x}{1+\sin x}-\frac{\cos^2 x}{1+\sin x}$

(1) $\frac{1+\sin x-\cos^2 x}{1+\sin x}=\frac{(1-\cos^2 x)+\sin x}{1+\sin x}$

$=\frac{\sin^2 x+\sin x}{1+\sin x}$

$=\frac{\sin x\,(\cancel{\sin x+1})}{\cancel{1+\sin x}}=\sin x$ (1)

The question can be answered in many methods

ii) ii) $(\sin x+\tan x)(\sin x-\tan x)$

(1) $\sin^2 x-\tan^2 x$

any (1)

$\tan^2 x\cos^2 x-\tan^2 x$

$\tan^2 x(\cos^2 x-1)$

$-\tan^2 x(1-\cos^2 x)$

(1) $-\tan^2 x\sin^2 x$

$\frac{\sin x}{\cos x}=\tan x$

$\sin x=\tan x\cos x$

$\sin^2 x=\tan^2 x\cos^2 x$

or $\sin^2 x-\tan^2 x$

$\sin^2 x-\frac{\sin^2 x}{\cos^2 x}$

$\frac{\sin^2 x\cos^2 x-\sin^2 x}{\cos^2 x}$

$\frac{\sin^2 x(\cos^2 x-1)}{\cos^2 x}$

6c. [3 marks] Show that:

$$(1 - \cos x)\left(1 + \frac{1}{\cos x}\right) = \tan x \sin x$$

1

$$\left(\frac{1-\cos x}{1}\right)\left(\frac{\cos x}{\cos x} + \frac{1}{\cos x}\right)$$

$$\frac{(1-\cos x)}{1} \cdot \left(\frac{(\cos x + 1)}{\cos x}\right)$$

1 $$\frac{1-\cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x} = \frac{\sin x \sin x}{\cos x}$$

1 $= \tan x \sin x$

$\boxed{\text{or}}$

$$(1-\cos x)\left(1+\frac{1}{\cos x}\right)$$

$$\frac{1-\cos x}{1} - \frac{\cos x}{\cos x} + \frac{1}{\cos x}$$

Let them have the same denomenator

$$\frac{\cos x}{\cos x} - \frac{\cos^2 x}{\cos x} + \frac{1-\cos x}{\cos x}$$

$$= \frac{\cos x - \cos^2 x + 1 - \cos x}{\cos x}$$

$$= \frac{1 - \cos^2 x + \cos x - \cos x}{\cos x}$$

$$= \frac{\sin^2 x + 0}{\cos x}$$

$$= \frac{\sin x \sin x}{\cos x}$$

$= \tan x \sin x$