REVIEW SET 11C

1 **a** $x \approx -6.1, -3.4$ **b** $x \approx 0.8$

2 **a** $x = \frac{3\pi}{2}$ **b** $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

3 **a** $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ **b** $x = -\pi, -\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

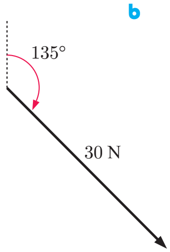
4 **a** $\cos\theta$ **b** $-\sin\theta$ **c** $5\cos^2\theta$ **d** $-\cos\theta$

5 **a** $4\sin^2\alpha - 4\sin\alpha + 1$ **b** $1 - \sin 2\alpha$

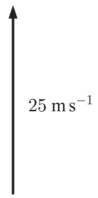
7 $\sin\theta = \frac{2}{\sqrt{13}}$, $\cos\theta = -\frac{3}{\sqrt{13}}$

EXERCISE 12A.1

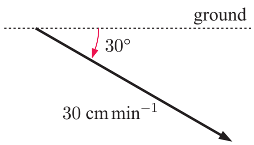
1 **a**



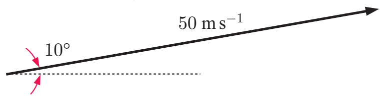
b



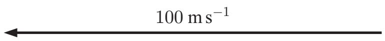
c



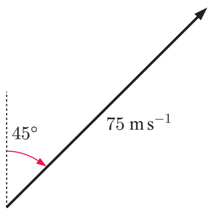
d



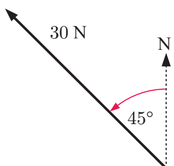
2 **a**



b

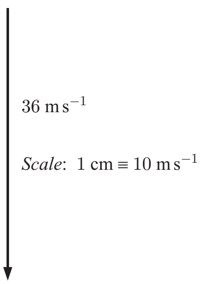


3 **a**

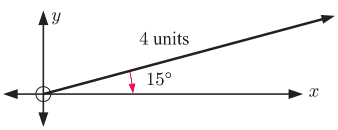


Scale: 1 cm ≡ 10 N

b



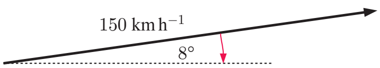
c



Scale: 1 cm ≡ 1 unit

d

Scale: 1 cm ≡ 30 km h^{-1}



EXERCISE 12A.2

1 **a** **p**, **q**, **s**, **t** **b** **p**, **q**, **r**, **t** **c** **p** and **r**, **q** and **t**
d **q**, **t** **e** **p** and **q**, **p** and **t**

2 **a** true **b** true **c** false **d** false **e** true **f** false

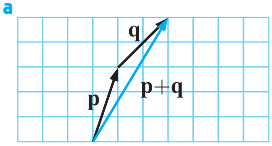
3 **a** **i** $\overrightarrow{BC}$ **ii** $\overrightarrow{ED}$

b **i** $\overrightarrow{FE}$, $\overrightarrow{BC}$
ii $\overrightarrow{DE}$, $\overrightarrow{EF}$, $\overrightarrow{FE}$, $\overrightarrow{FA}$, $\overrightarrow{AF}$, $\overrightarrow{AB}$, $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{DC}$

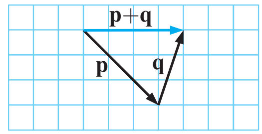
c $\overrightarrow{FC}$ (or $\overrightarrow{CF}$)

EXERCISE 12B.1

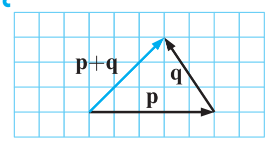
1 **a**



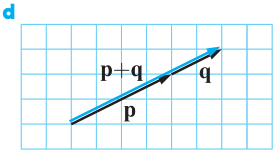
b



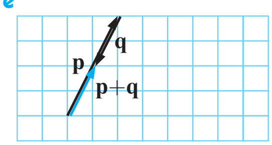
c



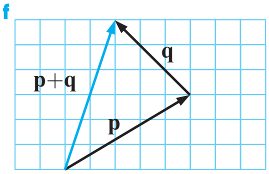
d



e

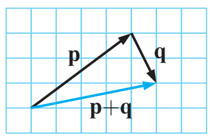


f

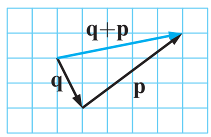


2 **a** $\overrightarrow{AC}$ **b** $\overrightarrow{BD}$ **c** **0** **d** $\overrightarrow{AD}$ **e** $\overrightarrow{AD}$ **f** **0**

3 **a** **i**

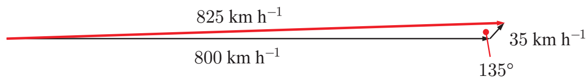


ii



b yes

5 **a** *Scale*: 1 cm ≡ 125 km

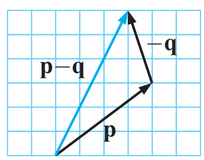


b We use vector addition.

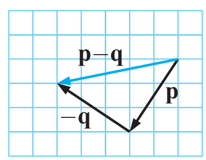
c 825 km h^{-1}, 88° east of north

EXERCISE 12B.2

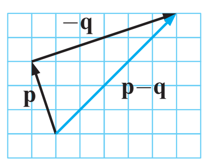
1 **a**



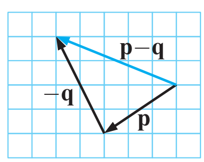
b



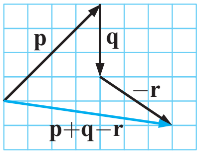
c



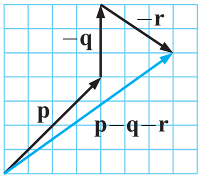
d



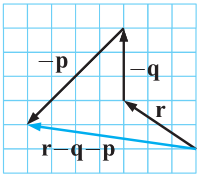
2 **a**



b



c



3 **a** $\overrightarrow{AB}$ **b** $\overrightarrow{AB}$ **c** $\mathbf{0}$ **d** $\overrightarrow{AD}$ **e** $\mathbf{0}$ **f** $\overrightarrow{AD}$

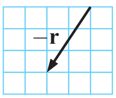
EXERCISE 12B.3

1 **a** $\mathbf{t} = \mathbf{r} + \mathbf{s}$ **b** $\mathbf{r} = -\mathbf{s} - \mathbf{t}$
c $\mathbf{r} = -\mathbf{p} - \mathbf{q} - \mathbf{s}$ **d** $\mathbf{r} = \mathbf{q} - \mathbf{p} + \mathbf{s}$
e $\mathbf{p} = \mathbf{t} + \mathbf{s} + \mathbf{r} - \mathbf{q}$ **f** $\mathbf{p} = -\mathbf{u} + \mathbf{t} + \mathbf{s} - \mathbf{r} - \mathbf{q}$

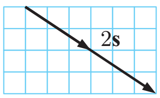
2 **a** **i** $\mathbf{r} + \mathbf{s}$ **ii** $-\mathbf{t} - \mathbf{s}$ **iii** $\mathbf{r} + \mathbf{s} + \mathbf{t}$
b **i** $\mathbf{p} + \mathbf{q}$ **ii** $\mathbf{q} + \mathbf{r}$ **iii** $\mathbf{p} + \mathbf{q} + \mathbf{r}$

EXERCISE 12B.4

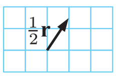
1 **a**



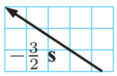
b



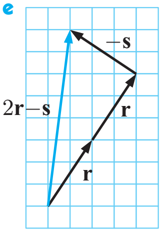
c



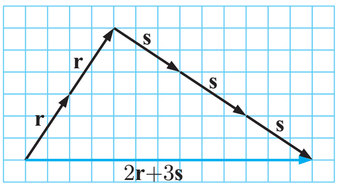
d



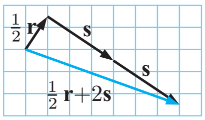
e



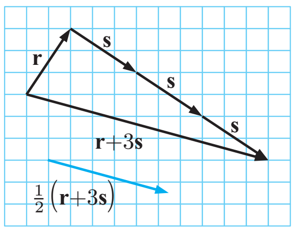
f



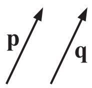
g



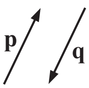
h



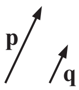
2 **a**



b



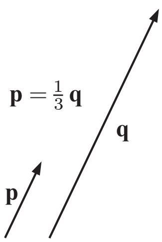
c



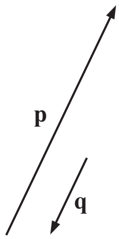
$\mathbf{p} = 2\mathbf{q}$

d

$\mathbf{p} = \frac{1}{3}\mathbf{q}$

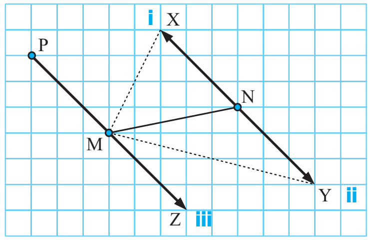


e



$\mathbf{p} = -3\mathbf{q}$

3 **a**



b a parallelogram

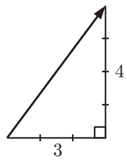
4 **a** $-\mathbf{p}$ **b** $\mathbf{p} + \mathbf{q}$ **c** $\frac{1}{2}(\mathbf{p} + \mathbf{q})$ **d** $\frac{1}{2}(\mathbf{q} - \mathbf{p})$

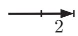
5 **a** $\mathbf{b}$ **b** $2\mathbf{b}$ **c** $\mathbf{b} - \mathbf{a}$ **d** $\mathbf{b} - \mathbf{a}$

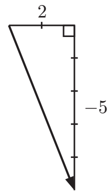
EXERCISE 12C

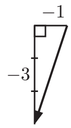
1 **a** $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$, $7\mathbf{i} + 3\mathbf{j}$ **b** $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$, $-6\mathbf{i}$
c $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$, $2\mathbf{i} - 5\mathbf{j}$ **d** $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$, $6\mathbf{j}$
e $\begin{pmatrix} -6 \\ 3 \end{pmatrix}$, $-6\mathbf{i} + 3\mathbf{j}$ **f** $\begin{pmatrix} -5 \\ -5 \end{pmatrix}$, $-5\mathbf{i} - 5\mathbf{j}$

2 **a** $3\mathbf{i} + 4\mathbf{j}$ **b** $2\mathbf{i}$ **c** $2\mathbf{i} - 5\mathbf{j}$ **d** $-\mathbf{i} - 3\mathbf{j}$



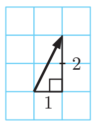


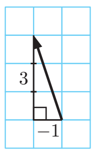


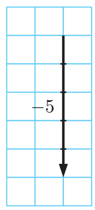


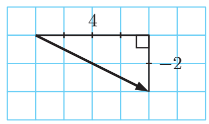
3 **a** $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$, $-4\mathbf{i} - \mathbf{j}$ **b** $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$, $-\mathbf{i} - 5\mathbf{j}$
c $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, $2\mathbf{i}$ **d** $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$, $3\mathbf{i} - 4\mathbf{j}$
e $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$, $-3\mathbf{i} + 4\mathbf{j}$ **f** $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $3\mathbf{i} + 5\mathbf{j}$

4 **a** $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ **b** $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ **c** $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$ **d** $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$









5 $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

EXERCISE 12D

1 **a** 5 units **b** 5 units **c** 2 units
d $\sqrt{8}$ units **e** 3 units

2 **a** $\sqrt{2}$ units **b** 13 units **c** $\sqrt{17}$ units
d 3 units **e** $|k|$ units

3 **a** unit vector **b** unit vector **c** not a unit vector
d unit vector **e** not a unit vector

4 **a** $k = \pm 1$ **b** $k = \pm 1$ **c** $k = 0$
d $k = \pm\frac{1}{\sqrt{2}}$ **e** $k = \pm\frac{\sqrt{3}}{2}$

5 $p = \pm 3$

EXERCISE 12E

1 **a** $\begin{pmatrix}-2\\6\end{pmatrix}$ **b** $\begin{pmatrix}-2\\6\end{pmatrix}$ **c** $\begin{pmatrix}-1\\-1\end{pmatrix}$ **d** $\begin{pmatrix}-1\\-1\end{pmatrix}$

e $\begin{pmatrix}-5\\-3\end{pmatrix}$ **f** $\begin{pmatrix}-5\\-3\end{pmatrix}$ **g** $\begin{pmatrix}-6\\4\end{pmatrix}$ **h** $\begin{pmatrix}-4\\1\end{pmatrix}$

2 **a** $\begin{pmatrix}-3\\7\end{pmatrix}$ **b** $\begin{pmatrix}-4\\-3\end{pmatrix}$ **c** $\begin{pmatrix}-8\\-1\end{pmatrix}$ **d** $\begin{pmatrix}-6\\9\end{pmatrix}$

e $\begin{pmatrix}0\\-5\end{pmatrix}$ **f** $\begin{pmatrix}6\\-9\end{pmatrix}$

3 **a** $\mathbf{a}+\mathbf{0}=\begin{pmatrix}a_1\\a_2\end{pmatrix}+\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}a_1+0\\a_2+0\end{pmatrix}=\begin{pmatrix}a_1\\a_2\end{pmatrix}=\mathbf{a}$

b $\mathbf{a}-\mathbf{a}=\begin{pmatrix}a_1\\a_2\end{pmatrix}-\begin{pmatrix}a_1\\a_2\end{pmatrix}=\begin{pmatrix}a_1-a_1\\a_2-a_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}=\mathbf{0}$

4 **a** $\begin{pmatrix}-3\\-15\end{pmatrix}$ **b** $\begin{pmatrix}-1\\2\end{pmatrix}$ **c** $\begin{pmatrix}0\\14\end{pmatrix}$ **d** $\begin{pmatrix}5\\-3\end{pmatrix}$

e $\begin{pmatrix}\frac{5}{2}\\\frac{11}{2}\end{pmatrix}$ **f** $\begin{pmatrix}-7\\7\end{pmatrix}$ **g** $\begin{pmatrix}5\\11\end{pmatrix}$ **h** $\begin{pmatrix}3\\\frac{17}{3}\end{pmatrix}$

5 **a** $\begin{pmatrix}8\\-1\end{pmatrix}$ **b** $\begin{pmatrix}8\\-1\end{pmatrix}$ **c** $\begin{pmatrix}8\\-1\end{pmatrix}$

6 **a** $\sqrt{13}$ units **b** $\sqrt{17}$ units **c** $5\sqrt{2}$ units **d** $\sqrt{10}$ units

e $\sqrt{29}$ units

7 **a** $\sqrt{10}$ units **b** $2\sqrt{10}$ units **c** $2\sqrt{10}$ units **d** $3\sqrt{10}$ units

e $3\sqrt{10}$ units **f** $2\sqrt{5}$ units **g** $8\sqrt{5}$ units **h** $8\sqrt{5}$ units

i $\sqrt{5}$ units **j** $\sqrt{5}$ units

EXERCISE 12F

1 **a** $\begin{pmatrix}2\\4\end{pmatrix}$ **b** $\begin{pmatrix}-2\\5\end{pmatrix}$ **c** $\begin{pmatrix}3\\-3\end{pmatrix}$ **d** $\begin{pmatrix}1\\-5\end{pmatrix}$

e $\begin{pmatrix}6\\-5\end{pmatrix}$ **f** $\begin{pmatrix}1\\3\end{pmatrix}$

2 **a** $(4, 2)$ **b** $(2, 2)$ **3** **a** $\begin{pmatrix}2\\1\end{pmatrix}$ **b** $(3, 3)$

4 **a** $\begin{pmatrix}5\\1\end{pmatrix}$ **b** $\begin{pmatrix}-5\\-1\end{pmatrix}$ **c** $D(-1, -2)$

5 **a** $\overrightarrow{AB}=\begin{pmatrix}4\\k-3\end{pmatrix}$, $|\overrightarrow{AB}|=\sqrt{16+(k-3)^2}=5$ units

b $k=0$ or 6

6 **a** $\overrightarrow{AB}=\begin{pmatrix}2\\3\end{pmatrix}$, $\overrightarrow{AC}=\begin{pmatrix}3\\-3\end{pmatrix}$

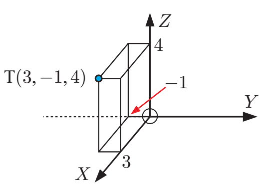
b $\overrightarrow{BC}=\overrightarrow{BA}+\overrightarrow{AC}=-\overrightarrow{AB}+\overrightarrow{AC}$ **c** $\overrightarrow{BC}=\begin{pmatrix}1\\-6\end{pmatrix}$

7 **a** $\begin{pmatrix}-5\\4\end{pmatrix}$ **b** $\begin{pmatrix}1\\2\end{pmatrix}$ **c** $\begin{pmatrix}6\\-5\end{pmatrix}$

8 **a** $M(1, 4)$ **b** $\overrightarrow{CA}=\begin{pmatrix}7\\5\end{pmatrix}$, $\overrightarrow{CM}=\begin{pmatrix}5\\3\end{pmatrix}$, $\overrightarrow{CB}=\begin{pmatrix}3\\1\end{pmatrix}$

EXERCISE 12G

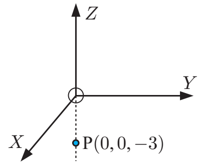
1 **a**



b $\overrightarrow{OT}=\begin{pmatrix}3\\-1\\4\end{pmatrix}$

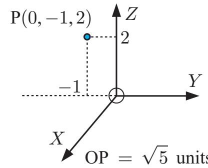
c $OT=\sqrt{26}$ units

2 **a**



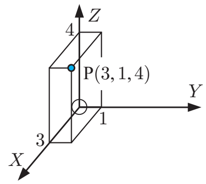
OP = 3 units

b

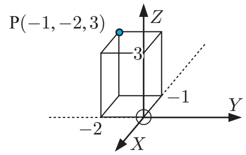


OP = $\sqrt{5}$ units

c



d



3 **a** $\overrightarrow{AB}=\begin{pmatrix}4\\-1\\-3\end{pmatrix}$, $\overrightarrow{BA}=\begin{pmatrix}-4\\1\\3\end{pmatrix}$

b $AB=\sqrt{26}$ units, $BA=\sqrt{26}$ units

4 $\overrightarrow{OA}=\begin{pmatrix}3\\1\\0\end{pmatrix}$, $\overrightarrow{OB}=\begin{pmatrix}-1\\1\\2\end{pmatrix}$, $\overrightarrow{AB}=\begin{pmatrix}-4\\0\\2\end{pmatrix}$

5 **a** $\overrightarrow{NM}=\begin{pmatrix}5\\-4\\-1\end{pmatrix}$ **b** $\overrightarrow{MN}=\begin{pmatrix}-5\\4\\1\end{pmatrix}$

c $MN=\sqrt{42}$ units

6 **a** $\overrightarrow{OA}=\begin{pmatrix}-1\\2\\5\end{pmatrix}$, $OA=\sqrt{30}$ units

b $\overrightarrow{AB}=\begin{pmatrix}3\\-2\\-2\end{pmatrix}$, $AB=\sqrt{17}$ units

c $\overrightarrow{AC}=\begin{pmatrix}-2\\-1\\-5\end{pmatrix}$, $AC=\sqrt{30}$ units

d $\overrightarrow{CB}=\begin{pmatrix}5\\-1\\3\end{pmatrix}$, $CB=\sqrt{35}$ units

e ABC is scalene, and not right angled.

7 **a** $\sqrt{13}$ units **b** $\sqrt{14}$ units **c** 3 units

9 **a** right angled **b** straight line (not a triangle)

10 **a** $|\overrightarrow{AB}|=\sqrt{158}$ units, $|\overrightarrow{BC}|=\sqrt{129}$ units,
$|\overrightarrow{AC}|=\sqrt{29}$ units, and $29+129=158$

b area ≈ 30.6 units^2

11 $(0, 3, 5)$, $r=\sqrt{3}$ units

12 **a** $(0, y, 0)$ **b** $(0, 2, 0)$ and $(0, -4, 0)$

13 **a** $a=5$, $b=6$, $c=-6$ **b** $a=4$, $b=2$, $c=1$

14 **a** $k=\pm\frac{\sqrt{11}}{4}$ **b** $k=\pm\frac{2}{3}$

15 **a** $r=2$, $s=4$, $t=-7$ **b** $r=-4$, $s=0$, $t=3$

16 **a** $\overrightarrow{AB}=\begin{pmatrix}2\\-5\\-1\end{pmatrix}$, $\overrightarrow{DC}=\begin{pmatrix}2\\-5\\-1\end{pmatrix}$

b ABCD is a parallelogram.

17 **a** $S(-2, 8, -3)$ **b** midpoints are at $(-\frac{1}{2}, 3, 1)$

EXERCISE 12H

1 **a** $\mathbf{x} = \frac{1}{2}\mathbf{q}$ **b** $\mathbf{x} = 2\mathbf{n}$ **c** $\mathbf{x} = -\frac{1}{3}\mathbf{p}$
d $\mathbf{x} = \frac{1}{2}(\mathbf{r} - \mathbf{q})$ **e** $\mathbf{x} = \frac{1}{5}(4\mathbf{s} - \mathbf{t})$ **f** $\mathbf{x} = 3(4\mathbf{m} - \mathbf{n})$

2 **a** $\mathbf{x} = \begin{pmatrix} 4 \\ -6 \\ -5 \end{pmatrix}$ **b** $\mathbf{x} = \begin{pmatrix} 1 \\ -\frac{2}{3} \\ \frac{5}{3} \end{pmatrix}$ **c** $\mathbf{x} = \begin{pmatrix} \frac{3}{2} \\ -1 \\ \frac{5}{2} \end{pmatrix}$

3 $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$, $AB = \sqrt{29}$ units

4 **a** $\overrightarrow{AB} = 4\mathbf{i} - 5\mathbf{j} - \mathbf{k}$ **b** $\sqrt{42}$ units

5 **a** $\sqrt{10}$ **b** $\sqrt{6}$ **c** $2\sqrt{10}$ **d** $2\sqrt{10}$
e $-3\sqrt{6}$ **f** $3\sqrt{6}$ **g** $3\sqrt{2}$ **h** $\sqrt{14}$

6 $\overrightarrow{AC} = -\mathbf{i} - 2\mathbf{k}$

8 C(5, 1, −8), D(8, −1, −13), E(11, −3, −18)

9 **a** parallelogram **b** parallelogram **c** not parallelogram

10 **a** D(9, −1) **b** R(3, 1, 6) **c** X(2, −1, 0)

11 **a** $\overrightarrow{BD} = \frac{1}{2}\mathbf{a}$ **b** $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ **c** $\overrightarrow{BA} = -\mathbf{b} + \mathbf{a}$
d $\overrightarrow{OD} = \mathbf{b} + \frac{1}{2}\mathbf{a}$ **e** $\overrightarrow{AD} = \mathbf{b} - \frac{1}{2}\mathbf{a}$ **f** $\overrightarrow{DA} = \frac{1}{2}\mathbf{a} - \mathbf{b}$

12 **a** $\begin{pmatrix} -1 \\ 5 \\ -1 \end{pmatrix}$ **b** $\begin{pmatrix} -3 \\ 4 \\ -2 \end{pmatrix}$ **c** $\begin{pmatrix} -3 \\ 6 \\ -5 \end{pmatrix}$

13 **a** $\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ **b** $\begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$ **c** $\begin{pmatrix} 1 \\ 4 \\ -9 \end{pmatrix}$
d $\begin{pmatrix} -1 \\ \frac{3}{2} \\ -\frac{7}{2} \end{pmatrix}$ **e** $\begin{pmatrix} 1 \\ -4 \\ 7 \end{pmatrix}$ **f** $\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$

14 **a** $\sqrt{11}$ units **b** $\sqrt{14}$ units **c** $\sqrt{38}$ units
d $\sqrt{3}$ units **e** $\begin{pmatrix} \sqrt{11} \\ -3\sqrt{11} \\ 2\sqrt{11} \end{pmatrix}$ **f** $\begin{pmatrix} -\frac{1}{\sqrt{11}} \\ \frac{1}{\sqrt{11}} \\ \frac{3}{\sqrt{11}} \end{pmatrix}$

15 **a** $a = \frac{1}{3}$, $b = 2$, $c = 1$ **b** $a = 1$, $b = -1$, $c = 2$
c $a = 4$, $b = -1$

EXERCISE 12I

1 $r = 3$, $s = -9$ **2** $a = -6$, $b = -4$

3 **a** $\overrightarrow{AB} \parallel \overrightarrow{CD}$, AB = 3 CD
b $\overrightarrow{RS} \parallel \overrightarrow{KL}$, RS = $\frac{1}{2}$KL opposite direction
c A, B, and C are collinear and AB = 2BC

4 **a** $\overrightarrow{PR} = \begin{pmatrix} -1 \\ -3 \\ 3 \end{pmatrix}$, $\overrightarrow{QS} = \begin{pmatrix} -2 \\ -6 \\ 6 \end{pmatrix}$, $2\overrightarrow{PR} = \overrightarrow{QS}$
b PR = $\frac{1}{2}$QS

5 **a** $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$ **b** $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$

6 **a** $\frac{1}{\sqrt{5}}(\mathbf{i} + 2\mathbf{j})$ **b** $\frac{1}{\sqrt{13}}(2\mathbf{i} - 3\mathbf{k})$ **c** $\frac{1}{3}(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$

7 **a** $\frac{3}{\sqrt{5}}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ **b** $\frac{2}{\sqrt{17}}\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

8 **a** $\overrightarrow{AB} = \begin{pmatrix} 2\sqrt{2} \\ -2\sqrt{2} \end{pmatrix}$ **b** $\overrightarrow{OB} = \begin{pmatrix} 3 + 2\sqrt{2} \\ 2 - 2\sqrt{2} \end{pmatrix}$
c $B(3 + 2\sqrt{2},\ 2 - 2\sqrt{2})$

9 **a** $\pm\frac{1}{3}\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ **b** $\pm\frac{2}{3}\begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$

10 **a** $\sqrt{2}\begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$ **b** $\frac{5}{3}\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

EXERCISE 12J

1 **a** 7 **b** 22 **c** 29 **d** 66 **e** 52 **f** 3 **g** 5 **h** 1

2 **a** 2 **b** 2 **c** 14 **d** 14 **e** 4 **f** 4

3 **a** −1 **b** 94.1° **4** **a** ≈ 140° **b** ≈ 114°

5 **a** 1 **b** 1 **c** 0 **6** **a** 5 **b** −9

7 **a** **i** ±12 **ii** 6
b **i** $\mathbf{a} \bullet \mathbf{b}$ is not 0 **ii** 12 units
c **i** $\mathbf{c} = \mathbf{d}$ **ii** $\mathbf{c} = -\mathbf{d}$

8 **a** $(\cos\theta, \sin\theta)$
b $\overrightarrow{BP} = \begin{pmatrix} \cos\theta + 1 \\ \sin\theta \end{pmatrix}$, $\overrightarrow{AP} = \begin{pmatrix} \cos\theta - 1 \\ \sin\theta \end{pmatrix}$
c $\overrightarrow{AP} \bullet \overrightarrow{BP} = \cos^2\theta + \sin^2\theta - 1$
$\therefore\ \overrightarrow{AP} \bullet \overrightarrow{BP} = 0$
d The angle in a semi-circle is a right angle.

10 **a** $t = 6$ **b** $t = -8$ **c** $t = 0$ or 2

11 **a** $t = -\frac{3}{2}$ **b** $t = -\frac{6}{7}$ **c** $t = \frac{-1 \pm \sqrt{5}}{2}$

12 **b** **Hint:** Show $\mathbf{a} \bullet \mathbf{b} = \mathbf{b} \bullet \mathbf{c} = \mathbf{a} \bullet \mathbf{c} = 0$
c **i** $t = -\frac{3}{2}$ **ii** $t = -\frac{5}{6}$

13 **a** $B\widehat{A}C$ is a right angle **b** not right angled
c $B\widehat{A}C$ is a right angle **d** $A\widehat{C}B$ is a right angle

14 $\overrightarrow{AB} \bullet \overrightarrow{AC} = 0$, $\therefore$ $B\widehat{A}C$ is a right angle

15 **b** $|\overrightarrow{AB}| = \sqrt{14}$ units, $|\overrightarrow{BC}| = \sqrt{14}$ units,
ABCD is a rhombus
c 0, the diagonals of a rhombus are perpendicular

16 **a** $k\begin{pmatrix} -2 \\ 5 \end{pmatrix}$, $k \neq 0$ **b** $k\begin{pmatrix} 2 \\ -1 \end{pmatrix}$, $k \neq 0$
c $k\begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $k \neq 0$ **d** $k\begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $k \neq 0$
e $k\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $k \neq 0$

17 **Hint:** Choose a vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$, where a and b are integers.
Solve for c such that $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = 0$.

18 $A\widehat{B}C \approx 62.5°$, the exterior angle $\approx 117.5°$

19 **a** 54.7° **b** 60° **c** 35.3°

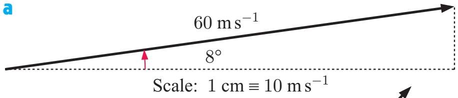
20 **a** 30.3° **b** 54.2° **21** **a** $M(\frac{3}{2}, \frac{5}{2}, \frac{3}{2})$ **b** 51.5°

22 **a** $t = 0$ or -3 **b** $r = -2$, $s = 5$, $t = -4$

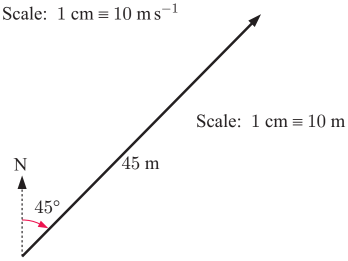
23 **a** 74.5° **b** 72.5°

REVIEW SET 12A

1 **a**



b



2 **a** $\overrightarrow{AC}$ **b** $\overrightarrow{AD}$

3 **a** $\mathbf{q} = \mathbf{p} + \mathbf{r}$ **b** $\mathbf{l} = \mathbf{k} - \mathbf{j} + \mathbf{n} - \mathbf{m}$ **4** $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

5 **a** $\mathbf{p} + \mathbf{q}$ **b** $\frac{3}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$ **6** $m = 5,\ n = -\frac{1}{2}$

7 $\begin{pmatrix} 8 \\ -8 \\ 7 \end{pmatrix}$ **8** **a** -13 **b** -36 **10** $k = 6$

11 $t\begin{pmatrix} 5 \\ 4 \end{pmatrix},\ t \neq 0$ **12** **a** **i** $\mathbf{p} + \mathbf{q}$ **ii** $\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$

13 **a** $\mathbf{a} \bullet \mathbf{b} = -4$ **b** $\mathbf{b} \bullet \mathbf{c} = 10$ **c** $\mathbf{a} \bullet \mathbf{c} = -10$

14 $a = -2,\ b = 0$

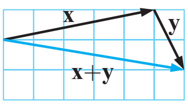
15 **a** $\mathbf{q} + \mathbf{r}$ **b** $\mathbf{r} + \mathbf{q}$, DB = AC, [DB] ∥ [AC]

16 **a** $t = -4$ **b** $\overrightarrow{LM} = \begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix}$, $\overrightarrow{KM} = \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}$

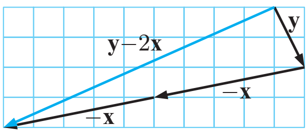
$\therefore\ \widehat{M} = 90°$

REVIEW SET 12B

1 **a**



b



2 AB = AC = $\sqrt{53}$ units and BC = $\sqrt{46}$ units

$\therefore$ △ is isosceles

3 **a** $\sqrt{13}$ units **b** $\sqrt{10}$ units **c** $\sqrt{109}$ units

4 $r = 4,\ s = 7$

5 **a** $\begin{pmatrix} -6 \\ 1 \\ 3 \end{pmatrix}$ **b** $\sqrt{46}$ units **c** $(-1, 3\frac{1}{2}, \frac{1}{2})$ **6** $c = \frac{50}{3}$

7 $\begin{pmatrix} 1 \\ -\frac{5}{3} \\ -\frac{2}{3} \end{pmatrix}$ **8** 64.0° **9** (0, 0, 1) and (0, 0, 9)

10 $t = \frac{2}{3}$ or -3 **11** **a** 8 **b** 62.2°

12 **a** $\overrightarrow{AC} = -\mathbf{p} + \mathbf{r}$, $\overrightarrow{BC} = -\mathbf{q} + \mathbf{r}$

13 $\pm\frac{4}{\sqrt{14}}(3\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ **14** 16.1°

15 **a** $k = \pm\frac{1}{2}$ **b** $-\frac{5}{\sqrt{14}}\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

REVIEW SET 12C

1 **a** $\overrightarrow{PQ}$ **b** $\overrightarrow{PR}$

2 **a** $\begin{pmatrix} 3 \\ -3 \\ 11 \end{pmatrix}$ **b** $\begin{pmatrix} 7 \\ -3 \\ -26 \end{pmatrix}$ **c** $\sqrt{74}$ units

3 **a** AB = $\frac{1}{2}$CD, [AB] ∥ [CD] **b** C is the midpoint of [AB].

4 **a** $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ 12 \\ 3 \end{pmatrix}$ **b** $\sqrt{162}$ units **c** $\sqrt{61}$ units

5 **a** $\mathbf{r} + \mathbf{q}$ **b** $-\mathbf{p} + \mathbf{r} + \mathbf{q}$ **c** $\mathbf{r} + \frac{1}{2}\mathbf{q}$ **d** $-\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{r}$

6 **a** $\mathbf{x} = \begin{pmatrix} -1 \\ \frac{1}{3} \\ \frac{2}{3} \end{pmatrix}$ **b** $\mathbf{x} = \begin{pmatrix} 1 \\ -10 \\ 2 \end{pmatrix}$ **7** $\mathbf{v} \bullet \mathbf{w} = \pm 6$

8 $t = 2 \pm \sqrt{2}$ **9** $\widehat{K} \approx 123.7°$, $\widehat{L} \approx 11.3°$, $\widehat{M} = 45°$

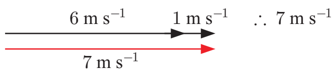
10 **a** $k = \pm\frac{12}{13}$ **b** $k = \pm\frac{1}{\sqrt{3}}$ **11** 40.7°

13 $\overrightarrow{OT} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$ or $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ **14** **a** 10 **b** 61.6°

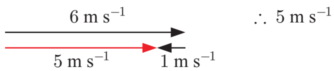
15 $\sin\theta = \frac{2}{\sqrt{5}}$

EXERCISE 13A

1 **a**



b



2 **a** 1.34 m s^{-1}

b **i** 30° to the right of straight across **ii** 1.04 m s^{-1}

3 **a** 24.6 km h^{-1} **b** ≈ 170°

4 **a** 82.5 m **b** 23.3° to the left of straight across **c** 48.4 s

5 **a** The plane's speed in still air would be ≈ 437 km h^{-1}.
The wind slows the plane down to 400 km h^{-1}.

b 4.65° north of due east

EXERCISE 13B

1 **a** **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} + t\begin{pmatrix} 1 \\ 4 \end{pmatrix},\ t \in \mathbb{R}$

ii $x = 3 + t,\ y = -4 + 4t,\ t \in \mathbb{R}$ **iii** $4x - y = 16$

b **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + t\begin{pmatrix} -2 \\ 5 \end{pmatrix},\ t \in \mathbb{R}$

ii $x = 5 - 2t,\ y = 2 + 5t,\ t \in \mathbb{R}$ **iii** $5x + 2y = 21$

c **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \end{pmatrix} + t\begin{pmatrix} 3 \\ 7 \end{pmatrix},\ t \in \mathbb{R}$

ii $x = -6 + 3t,\ y = 7t,\ t \in \mathbb{R}$ **iii** $7x - 3y = -42$

d **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 11 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \end{pmatrix},\ t \in \mathbb{R}$

ii $x = -1 - 2t,\ y = 11 + t,\ t \in \mathbb{R}$ **iii** $x + 2y = 21$

2 **a** $x = -1 + 2t,\ y = 4 - t,\ t \in \mathbb{R}$

b Points are: $(-1, 4), (1, 3), (5, 1), (-3, 5), (-9, 8)$

3 **a** When $t = 1$, $x = 3$, $y = -2$, $\therefore$ yes **b** $k = -5$

4 **a** (0, 8) **b** It is a non-zero scalar multiple of $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

c $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \end{pmatrix} + s\begin{pmatrix} 1 \\ -3 \end{pmatrix},\ s \in \mathbb{R}$

5 **a** **i** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \ t \in \mathbb{R}$

ii $x = 1 + 2t, \ y = 3 + t, \ z = -7 + 3t, \ t \in \mathbb{R}$

b **i** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \ t \in \mathbb{R}$

ii $x = 6, \ y = 1 + t, \ z = 2 - 2t, \ t \in \mathbb{R}$

c **i** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \ t \in \mathbb{R}$

ii $x = -2 + t, \ y = 2, \ z = 1, \ t \in \mathbb{R}$

d **i** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \ t \in \mathbb{R}$

ii $x = 2t, \ y = 2 - t, \ z = -1 + 3t, \ t \in \mathbb{R}$

6 **a** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \ t \in \mathbb{R}$

b $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}, \ t \in \mathbb{R}$

c $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} + t\begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}, \ t \in \mathbb{R}$

d $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + t\begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix}, \ t \in \mathbb{R}$

7 **a** $(-\frac{1}{2}, \frac{9}{2}, 0)$ **b** $(0, 4, 1)$ **c** $(4, 0, 9)$

8 $(0, 7, 3)$ and $(\frac{20}{3}, -\frac{19}{3}, -\frac{11}{3})$

EXERCISE 13C

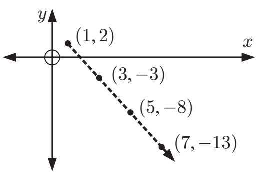
1 33.7° **2** $\mathbf{b}_1 \bullet \mathbf{b}_2 = 0$ **3** 75.5°

4 **a** 28.6° **b** $x = -\frac{48}{7}$

5 **a** 78.7° **b** 63.4° **c** 63.4° **d** 71.6°

EXERCISE 13D

1 **a** $(1, 2)$ **b**



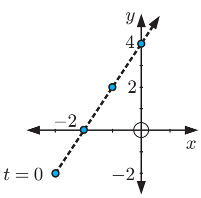
c $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$

d $\sqrt{29}$ cm s^{-1}

2 **a** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 4 \\ -5 \end{pmatrix}, \ t \geqslant 0$ **b** $(8, -4.5)$

c 45 minutes

3 **a** $\begin{pmatrix} -3 + 2t \\ -2 + 4t \end{pmatrix}$ **d**



b $\begin{pmatrix} 2 \\ 8 \end{pmatrix}$

c **i** $t = 1.5$ s

ii $t = 0.5$ s

4 **a** **i** $(-4, 3)$ **ii** $\begin{pmatrix} 12 \\ 5 \end{pmatrix}$ **iii** 13 m s^{-1}

b **i** $(3, 0, 4)$ **ii** $\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ **iii** 3 m s^{-1}

5 **a** $\begin{pmatrix} 120 \\ -90 \end{pmatrix}$ **b** $\begin{pmatrix} 20\sqrt{5} \\ 10\sqrt{5} \end{pmatrix}$ **6** $\begin{pmatrix} -12 \\ 30 \\ -84 \end{pmatrix}$

7 **a** A is at $(4, 5)$, B is at $(1, -8)$

b For A it is $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$. For B it is $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

c For A, speed is $\sqrt{5}$ km h^{-1}. For B, speed is $\sqrt{5}$ km h^{-1}.

d $\begin{pmatrix} 1 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 0$

8 **a** $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$\therefore \ x_1(t) = -5 + 3t, \ \ y_1(t) = 4 - t$

b speed $= \sqrt{10}$ km min^{-1}

c a minutes later, $(t - a)$ min have elapsed.

$\therefore \ \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 15 \\ 7 \end{pmatrix} + (t - a)\begin{pmatrix} -4 \\ -3 \end{pmatrix}$

$\therefore \ x_2(t) = 15 - 4(t - a), \ \ y_2(t) = 7 - 3(t - a)$

d Torpedo is fired at 1:35:28 pm and the explosion occurs at 1:37:42 pm.

9 **a** $\begin{pmatrix} -3 \\ 1 \\ -0.5 \end{pmatrix}$ **b** ≈ 19.2 km h^{-1}

c $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 1 \\ -0.5 \end{pmatrix}, \ t \in \mathbb{R}$ **d** 1 hour

EXERCISE 13E

1 **a** $\frac{3}{5}\sqrt{5}$ units **b** $\frac{1}{2}\sqrt{2}$ units **c** $\frac{9}{2}\sqrt{2}$ units

d 0 units

2 **a** $6\mathbf{i} - 6\mathbf{j}$ **b** $\begin{pmatrix} 6 - 6t \\ -6 + 8t \end{pmatrix}$

c when $t = \frac{3}{4}$ hours

d $t = 0.84$ and position is $(0.96, 0.72)$

3 **a** $\begin{pmatrix} -120 \\ -40 \end{pmatrix}$ **b** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 200 \\ 100 \end{pmatrix} + t\begin{pmatrix} -120 \\ -40 \end{pmatrix}$

c $(80, 60)$ **d** $|\overrightarrow{OP}| = \sqrt{80^2 + 60^2} = 100$ km

e at 1:45 pm and $d_{min} \approx 31.6$ km **f** 2:30 pm

4 **a** A$(18, 0)$ and B$(0, 12)$ **b** R is at $\left(x, \dfrac{36 - 2x}{3}\right)$

c $\overrightarrow{PR} = \begin{pmatrix} x - 4 \\ \frac{36 - 2x}{3} \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} -18 \\ 12 \end{pmatrix}$

d $(\frac{108}{13}, \frac{84}{13})$ and distance ≈ 7.77 km

5 **a** A$(3, -4)$ and B$(4, 3)$

b for A $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$, for B $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ **c** 82.9°

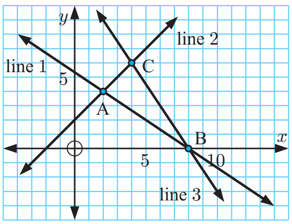
d at $t = 1.5$ hours

6 **a** $(2, -1, 4)$ **b** $\sqrt{27}$ units

7 **a** $(2, \frac{1}{2}, \frac{5}{2})$ **b** $\sqrt{\frac{3}{2}}$ units

EXERCISE 13F

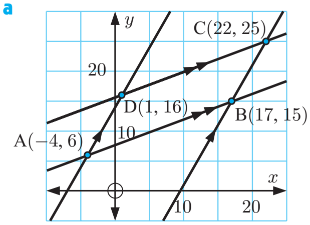
1 **a**



b A(2, 4), B(8, 0), C(4, 6)

c BC = BA $= \sqrt{52}$ units $\therefore$ isosceles $\triangle$

2 **a**



b A(−4, 6), B(17, 15), C(22, 25), D(1, 16)

3 **a** A(2, 3), B(8, 6), C(5, 0)

b $AB = BC = \sqrt{45}$ units, $AC = \sqrt{18}$ units

4 **a** P(10, 4), Q(3, −1), R(20, −10)

b $\overrightarrow{PQ} = \begin{pmatrix} -7 \\ -5 \end{pmatrix}$, $\overrightarrow{PR} = \begin{pmatrix} 10 \\ -14 \end{pmatrix}$, $\overrightarrow{PQ} \bullet \overrightarrow{PR} = 0$

c $Q\widehat{P}R = 90°$ **d** 74 units^2

5 **a** A is at (2, 5), B(18, 9), C(14, 25), D(−2, 21)

b $\overrightarrow{AC} = \begin{pmatrix} 12 \\ 20 \end{pmatrix}$ and $\overrightarrow{DB} = \begin{pmatrix} 20 \\ -12 \end{pmatrix}$

i $\sqrt{544}$ units **ii** $\sqrt{544}$ units **iii** 0

c Diagonals are perpendicular and equal in length, and as their midpoints are both (8, 15), ABCD is a square.

EXERCISE 13G

1 **a** They intersect at (1, 2, 3), angle $\approx 10.9°$.

b Lines are skew, angle $\approx 62.7°$.

c They are parallel, $\therefore$ angle $= 0°$.

d They are skew, angle $\approx 11.4°$.

e They intersect at (−4, 7, −7), angle $\approx 40.2°$.

f They are parallel, $\therefore$ angle $= 0°$.

g They are coincident, $\therefore$ angle $= 0°$

REVIEW SET 13A

1 **a** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

b $x = -6 + 4t$, $y = 3 - 3t$, $t \in \mathbb{R}$ **c** $3x + 4y = -6$

2 $m = 10$

3 **a** (5, 2) **b** $\begin{pmatrix} 4 \\ 10 \end{pmatrix}$ is a non-zero scalar multiple of $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$

c $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + s\begin{pmatrix} 4 \\ 10 \end{pmatrix}$

4 **a** $x = 2 + t$, $y = 4t$, $z = 1 - 3t$, $t \in \mathbb{R}$

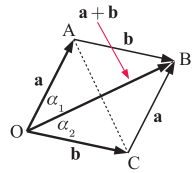
b Use $\cos\theta = \dfrac{|\overrightarrow{PQ} \bullet \overrightarrow{QR}|}{|\overrightarrow{PQ}||\overrightarrow{QR}|}$

5 **a** A(5, 2), B(6, 5), C(8, 3)

b $|\overrightarrow{AB}| = \sqrt{10}$ units, $|\overrightarrow{BC}| = \sqrt{8}$ units, $|\overrightarrow{AC}| = \sqrt{10}$ units

c isosceles

6 **a** OABC is a rhombus. So, its diagonals bisect its angles.

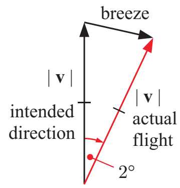


b $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \\ -4 \end{pmatrix} + t\begin{pmatrix} 0 \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$, $t \in \mathbb{R}$

c $(7, 3\frac{3}{4}, -3\frac{1}{4})$

7 (4, 1, −3) and (1, −5, 0)

8 **a**



b **i** isosceles triangle $\therefore$ 2 remaining angles $= 89°$ each, breeze makes angle of $180 - 89 = 91°$ to intended direction of the arrow.

ii bisect angle 2° and use $\sin 1° = \dfrac{\frac{1}{2}\text{ speed}}{|\mathbf{v}|}$

$\therefore$ speed $= 2\,|\mathbf{v}|\sin 1°$

REVIEW SET 13B

1 $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \end{pmatrix} + t\begin{pmatrix} 5 \\ 4 \end{pmatrix}$, $t \in \mathbb{R}$

2 **a** **i** $-6\mathbf{i} + 10\mathbf{j}$ **ii** $-5\mathbf{i} - 15\mathbf{j}$

iii $(-6 - 5t)\mathbf{i} + (10 - 15t)\mathbf{j}$

b $t = 0.48$ h

c shortest distance ≈ 8.85 km, so will miss reef

3 **a** **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} + t\begin{pmatrix} 4 \\ -1 \end{pmatrix}$, $t \in \mathbb{R}$

ii $x = 2 + 4t$, $y = -3 - t$, $t \in \mathbb{R}$

b **i** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \\ 3 \end{pmatrix} + t\begin{pmatrix} 6 \\ -8 \\ -3 \end{pmatrix}$, $t \in \mathbb{R}$

ii $x = -1 + 6t$, $y = 6 - 8t$, $z = 3 - 3t$, $t \in \mathbb{R}$

4 **a** 11.5° east of due north **b** ≈ 343 km h^{-1} **5** 8.13°

6 **a** X23, $x_1 = 2 + t$, $y_1 = 4 - 3t$, $t \geqslant 0$

b Y18, $x_2 = 13 - t$, $y_2 = 3 - 2a + at$, $t \geqslant 2$

c interception occurred at 2:22:30 pm

d bearing $\approx 193°$, ≈ 4.54 units per minute

7 **a** intersecting at (4, 3, 1), angle $\approx 44.5°$

b skew, angle $\approx 71.2°$

REVIEW SET 13C

1 $2\sqrt{10}(3\mathbf{i} - \mathbf{j})$

2 **a** (−4, 3) **b** (28, 27) **c** $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$ **d** 10 m s^{-1}

3 **a** (KL) is parallel to (MN) as $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ is parallel to $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$

b (KL) is perpendicular to (NK) as $\begin{pmatrix} 5 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 10 \end{pmatrix} = 0$

and (NK) is perpendicular to (MN) as $\begin{pmatrix} 4 \\ 10 \end{pmatrix} \bullet \begin{pmatrix} -5 \\ 2 \end{pmatrix} = 0$

c K(7, 17), L(22, 11), M(33, −5), N(3, 7) **d** 261 units^2

4 30.5°

5 **a** $|\overrightarrow{AB}| = \sqrt{22}$ units

b A lies on the line **r** where $\lambda = -3$ and B lies on **r** where $\lambda = 0$ $\therefore$ the line between A and B is the same as line **r**, so it can be described by **r**.

c 70.5°

6 **a** Road A: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -9 \\ 2 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix}$, $t \in \mathbb{R}$

Road B: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ -18 \end{pmatrix} + s\begin{pmatrix} 5 \\ 12 \end{pmatrix}$, $s \in \mathbb{R}$

b Road B, 13 km

7 **a** $\overrightarrow{AB} \bullet \overrightarrow{AC} = \begin{pmatrix} -2 \\ -1 \\ 6 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix} = 0$

b **i** $x = 4 - 2t$, $y = 2 - t$, $z = -1 + 6t$, $t \in \mathbb{R}$

ii $x = 4 + 5s$, $y = 2 + 2s$, $z = -1 + 2s$, $s \in \mathbb{R}$

EXERCISE 14A

1 **a** 7 **b** 7 **c** 11 **d** 16 **e** 0 **f** 5

2 **a** 5 **b** 7 **c** c

3 **a** −2 **b** 7 **c** −1 **d** 1

4 **a** −3 **b** 5 **c** −1 **d** 6 **e** −4 **f** −8 **g** 1 **h** 2 **i** 5

EXERCISE 14B

1 **a** **i** as $x \to 0^-$, $f(x) \to -\infty$
as $x \to 0^+$, $f(x) \to \infty$
as $x \to \infty$, $f(x) \to 0^+$
as $x \to -\infty$, $f(x) \to 0^-$
vertical asymptote $x = 0$, horizontal asymptote $y = 0$

ii $\lim_{x\to-\infty} f(x) = 0$, $\lim_{x\to\infty} f(x) = 0$

b **i** as $x \to -3^-$, $f(x) \to \infty$
as $x \to -3^+$, $f(x) \to -\infty$
as $x \to \infty$, $f(x) \to 3^-$
as $x \to -\infty$, $f(x) \to 3^+$
vertical asymptote $x = -3$, horizontal asymptote $y = 3$

ii $\lim_{x\to-\infty} f(x) = 3$, $\lim_{x\to\infty} f(x) = 3$

c **i** as $x \to -\frac{2}{3}^-$, $f(x) \to -\infty$
as $x \to -\frac{2}{3}^+$, $f(x) \to \infty$
as $x \to \infty$, $f(x) \to -\frac{2}{3}^+$
as $x \to -\infty$, $f(x) \to -\frac{2}{3}^-$
vertical asymptote $x = -\frac{2}{3}$,
horizontal asymptote $y = -\frac{2}{3}$

ii $\lim_{x\to-\infty} f(x) = -\frac{2}{3}$, $\lim_{x\to\infty} f(x) = -\frac{2}{3}$

d **i** as $x \to 1^-$, $f(x) \to \infty$,
as $x \to 1^+$, $f(x) \to -\infty$,
as $x \to \infty$, $f(x) \to -1^-$
as $x \to -\infty$, $f(x) \to -1^+$
vertical asymptote $x = 1$, horizontal asymptote $y = -1$

ii $\lim_{x\to-\infty} f(x) = -1$, $\lim_{x\to\infty} f(x) = -1$

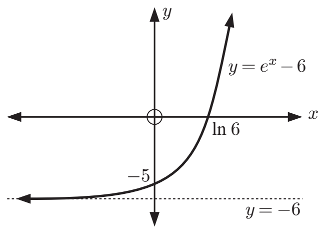
e **i** as $x \to \infty$, $y \to 1^-$ horizontal asymptote $y = 1$
as $x \to -\infty$, $y \to 1^-$ no vertical asymptote

ii $\lim_{x\to-\infty} f(x) = 1$, $\lim_{x\to\infty} f(x) = 1$

f **i** as $x \to \infty$, $f(x) \to 0^+$
as $x \to -\infty$, $f(x) \to 0^-$
horizontal asymptote $y = 0$
no vertical asymptote

ii $\lim_{x\to-\infty} f(x) = 0$, $\lim_{x\to\infty} f(x) = 0$

2 **a**



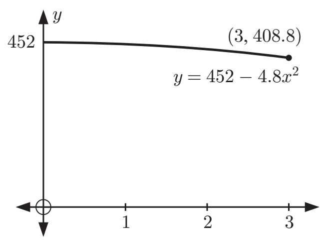
b **i** $\lim_{x\to-\infty} (e^x - 6) = -6$

ii $\lim_{x\to\infty} (e^x - 6)$ does not exist
$y = -6$ is a horizontal asymptote of $y = e^x - 6$.

3 $\lim_{x\to-\infty} (2e^{-x} - 3)$ does not exist, $\lim_{x\to\infty} (2e^{-x} - 3) = -3$

EXERCISE 14C

1 **a**



b No

c **i** 0 m s^{-1} **ii** 9.6 m s^{-1} **iii** 19.2 m s^{-1} **iv** 28.8 m s^{-1}

2 **a**

x	Point B	Gradient of [AB]
0	(0, 0)	2
1	(1, 1)	3
1.5	(1.5, 2.25)	3.5
1.9	(1.9, 3.61)	3.9
1.99	(1.99, 3.9601)	3.99
1.999	(1.999, 3.996 001)	3.999

x	Point B	Gradient of [AB]
5	(5, 25)	7
3	(3, 9)	5
2.5	(2.5, 6.25)	4.5
2.1	(2.1, 4.41)	4.1
2.01	(2.01, 4.0401)	4.01
2.001	(2.001, 4.004 001)	4.001

b $\lim_{x\to2} \dfrac{x^2-4}{x-2} = 4$
The gradient of the tangent to $y = x^2$ at the point (2, 4) is 4.

EXERCISE 14D

1 **a** 3 **b** 0 **2** **a** 4 **b** −1 **3** $f(2) = 3$, $f'(2) = 1$

EXERCISE 14E

1 **a** **i** 1 **ii** 0 **iii** $3x^2$ **iv** $4x^3$ **b** $f'(x) = nx^{n-1}$

2 **a** 2 **b** $2x - 3$ **c** $-2x + 5$

3 **a** $\dfrac{dy}{dx} = -1$ **b** $\dfrac{dy}{dx} = 4x + 1$ **c** $\dfrac{dy}{dx} = 3x^2 - 4x$