Teachers Teaching with Technology

T^3 Scotland

T^3 EUROPE

Functions and Transformations

Including

Functions and Transformations Challenge

FUNCTIONS AND TRANSFORMATIONS

Aim

The aim of this module is to investigate the effect of the transformation of
$y = f(x)$ to $y = f(x) + k$, $y = k\,f(x)$, $y = f(x + k)$, $y = f(kx)$
or any combination of these, where **k** is a non-zero real number.

Objectives

Mathematical objectives

By the end of this unit you should be able to:

- Show and explain the translation of any function by any of the transformations above.
- Explain the effect of combining transformations.

Calculator objectives

By the end of this unit you should be able to:

- Apply transformation to functions via [Y=].
- Draw graphs of all functions using appropriate settings.
- Define a function in terms of one already defined. (Hint Sheet Number 5)

STUDENT TASK

1. Read the Calculator Skills Sheet (page 3) carefully before you start, it may prevent you encountering difficulties with your TI-83.

2. On the worksheets (Pages 4 to 8), for each pair of equations you must:
   i. sketch the graphs obtained using the TI-83 in two colours on the axis.
   ii. complete the statements based on your observations.
   iii. make a Generalisation bsed upon your observations.

3. On the Pages 9-12 you are to put down your TI-83 and try to complete the transformations required.

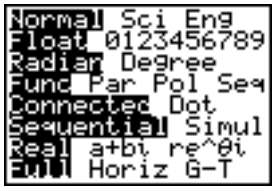
4 Finally, can you generate a graph for a friend to transform.
Using the window range given on Skills Sheet Item 4, enter a function on the Y= screen and get a friend to draw transformations of this function on the grids given on page 12. You can check the answers using the TI-83.

FUNCTIONS AND TRANSFORMATIONS

Calculator skills sheet

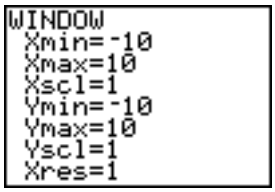
Before we can start on this unit of work we must first ensure that your TI-83 is in the correct MODE, and is going to operate as we want it to.
This is how we do this

1. Press the [MODE] button.
   The display should look exactly like this.
   If it does not look like this, then using the cursor keys highlight the correct item in each line and press [ENTER] to change the selection.
   Notice: There can only be one item in each line highlighted.

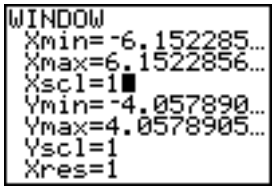


2. Press the [2 nd] and [FORMAT].
   This takes you to the WINDOW FORMAT screen.
   It should look like this.
   If it does not then using the cursor keys highlight the correct item in each line and press [ENTER].
   Once the screen looks like this press [CLEAR].
   Notice: There can only be one item in each line highlighted.

3. Depended upon the type of function being graphed you must choose an appropriate WINDOW range.
   [ZOOM] [6] or [ZOOM] [4] are both useful for polynomial type functions, whereas [ZOOM] [7] is useful for trigonometric functions.



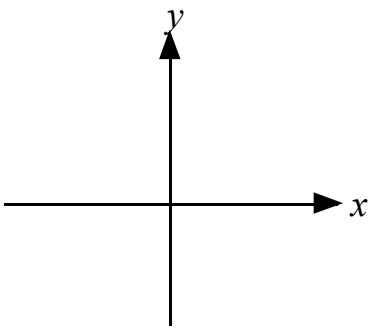
4. To get the same window range as has been used in Pages 10 - 12, follow this procedure. First ensue that the MODE screen is set to Radians, then [ZOOM] [7] followed by [ZOOM] [5].
   Once this is done on the WINDOW screen adjust Xscl = 1.
   This gives a range that is good for many varied graph types.



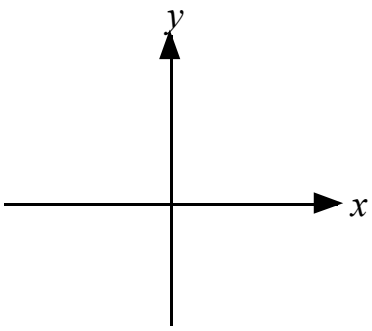
Task 1

If you know the graph of $y = x^2$ can you draw quickly the graph of $y = x^2 + 5$?

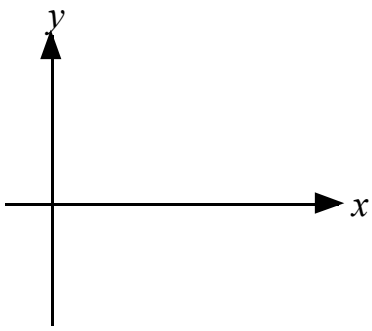
On your calculator draw these two graphs and *sketch* the results (two colours) on the axes below.



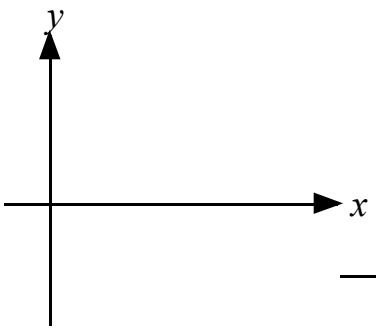
- $y = x^2$
- $y = x^2 + 5$



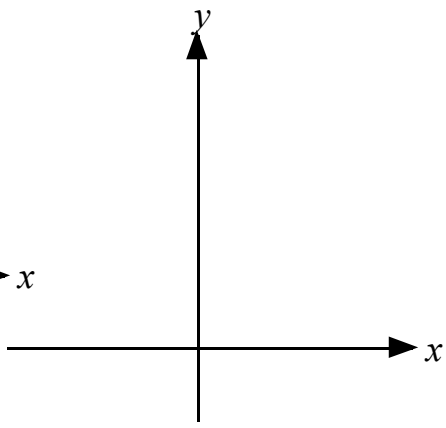
- $y = x^3$
- $y = x^3 - 5$



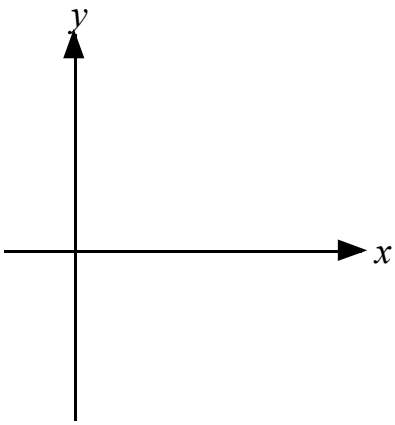
- $y = \cos(x)$
- $y = \cos(x) + 3$



- $y = \sin(x) - 3$
- $y = \sin(x) - 3$



- $y = 2^x$
- $y = 2^x + 5$



- $y = x^3 + x + 3$
- $y = x^3 + x + 3 + 2$

or $y = x^3 + x + 5$

Complete each of these statements:

Conclusion

Given the graph of $y = f(x)$ to draw the graph of $y = f(x) - 3$

you_____

_____

General conclusion

Given the graph of $y = f(x)$ to draw the graph of $y = f(x) + \boldsymbol{k}$

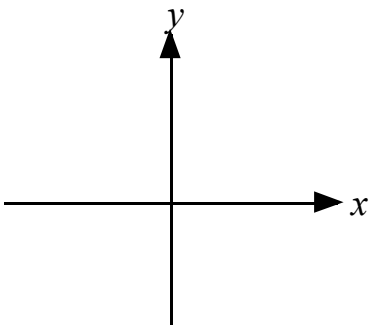
you_____

_____

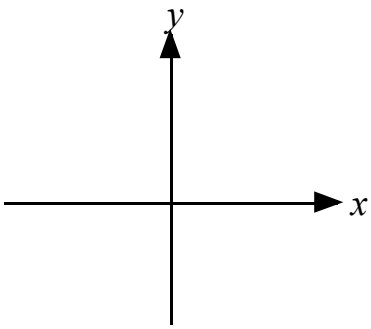
Task 2

If you know the graph of $y = x^2$ can you draw quickly the graph of $y = -x^2$?

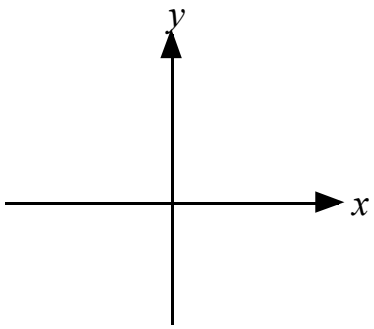
On your calculator draw these two graphs and *sketch* the results (two colours) on the axes below.



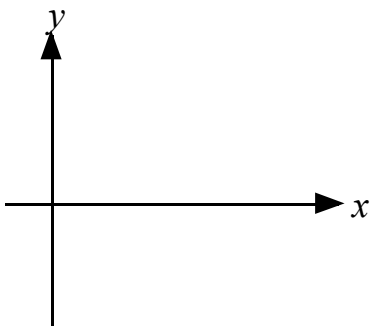
- $y = x^2$
- $y = -x^2$



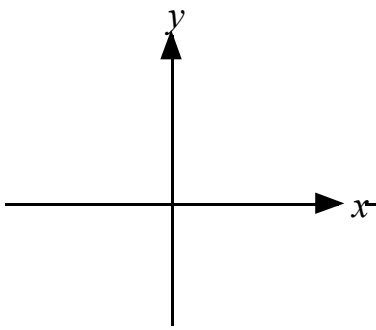
- $y = x^3 + 5$
- $y = -(x^3 + 5)$



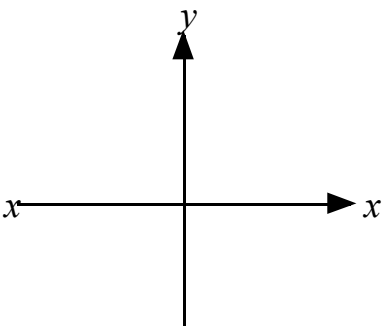
- $y = x$
- $y = -x$



- $y = \sin(x)$
- $y = -\sin(x)$



- $y = 2^x$
- $y = -2^x$



- $y = x^2 - 4x + 7$
- $y = -(x^2 - 4x + 7)$

Complete this statement:

General conclusion

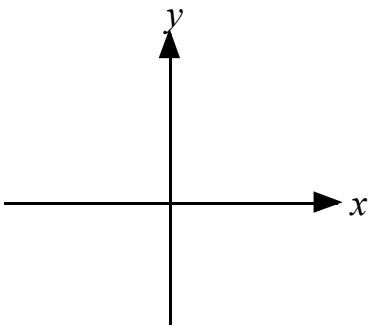
Given the graph of $y = f(x)$ to draw the graph of $y = -f(x)$ you_____

_____

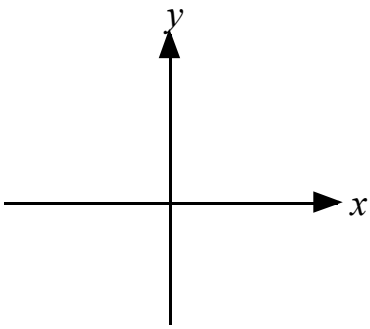
Task 3

If you know the graph of $y = x^2$ can you draw quickly the graph of $y = \boldsymbol{k}x^2$?

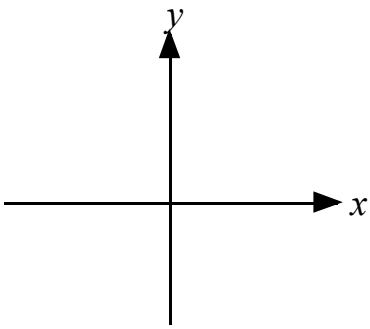
Let's take $\boldsymbol{k} = 3$, using your calculator draw the following functions and *sketch* the results (two colours) on the axes below.



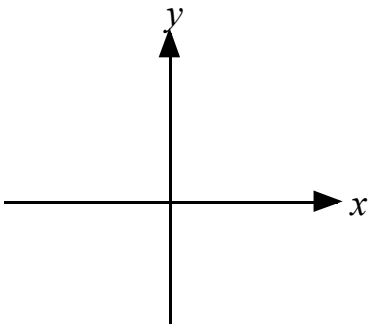
- $y = x^2$
- $y = 3x^2$



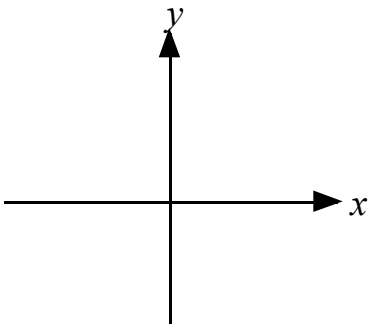
- $y = x^2 - 2$
- $y = 3(x^2 - 2)$



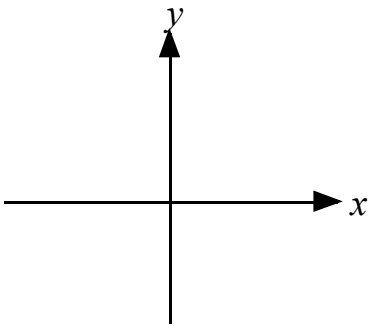
- $y = x^2 + 1$
- $y = 3(x^2 + 1)$



- $y = \sin(x)$
- $y = 3\sin(x)$



- $y = x$
- $y = 3x$



- $y = x^3 + 3x^2 - 2$
- $y = 3(x^3 + 3x^2 - 2)$

Complete each of these statements:

Conclusion

Given the graph of $y = f(x)$ to draw the graph of $y = 3f(x)$ you_____

_____

General conclusion

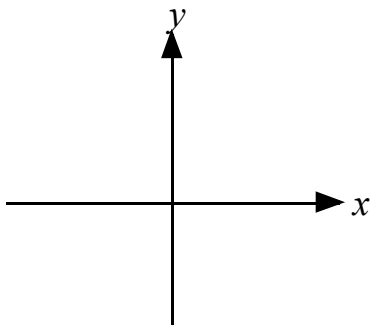
Given the graph of $y = f(x)$ to draw the graph of $y = \boldsymbol{k}f(x)$ you_____

_____

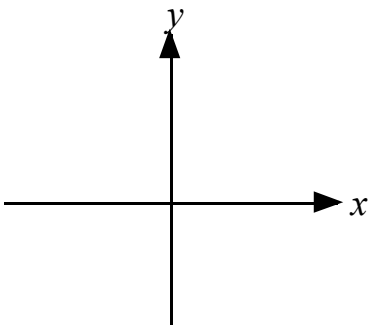
Task 4

If you know the graph of $y = x^2$ can you draw quickly the graph of $y = (x + 3)^2$?

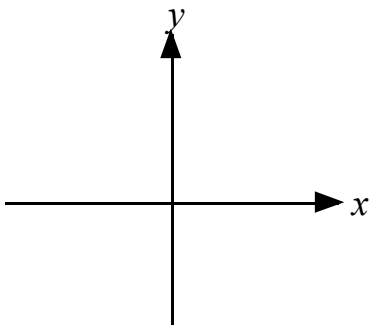
On your calculator draw these two graphs and *sketch* (two colours) the results on the axes below.



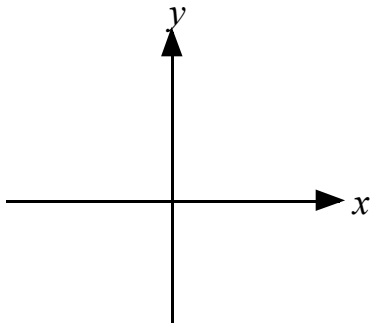
- $y = x^2$
- $y = (x + 3)^2$



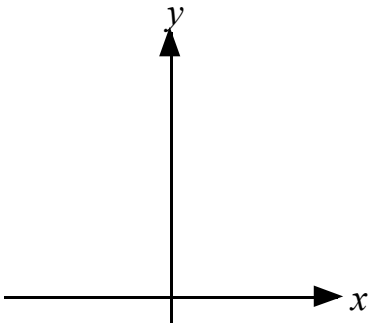
- $y = x$
- $y = x + 3$



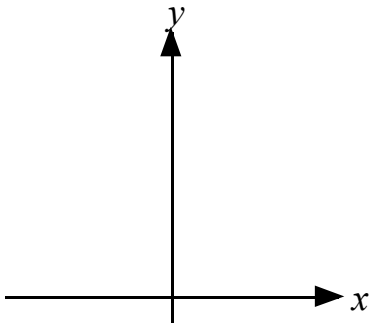
- $y = \sin(x)$
- $y = \sin(x + 30)$



- $y = x^2 - 5$
- $y = (x + 3)^2 - 5$



- $y = x^2 + 4$
- $y = (x + 3)^2 + 4$



- $y = x^2 - 2x + 4$
- $y = (x+3)^2 - 2(x+3) + 4$

Complete this statement:

General conclusion

Given the graph of $y = f(x)$ to draw the graph of $y = f(x + \boldsymbol{k})$ you_____

_____

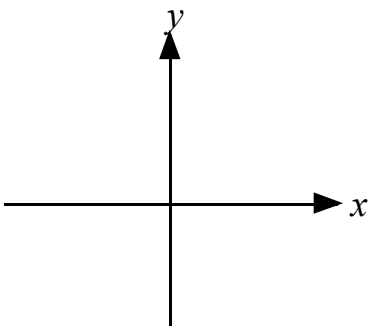
_____

Task 5

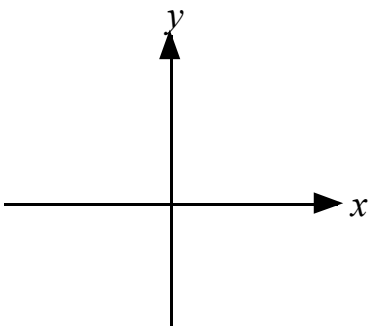
If you know the graph of $y = x^2$ can you draw quickly the graph of $y = (-x)^2$.

On your calculator draw these two graphs and *sketch* (two colours) the results on the axes below.

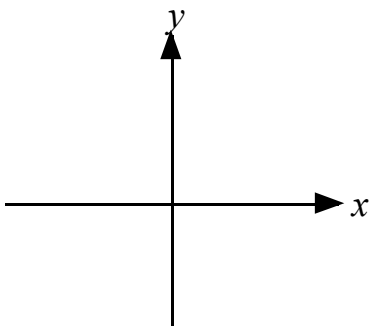
Do the same for the other pairs of graphs.



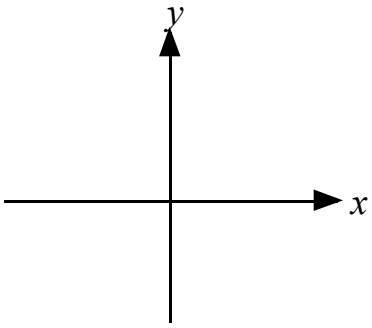
- $y = x^3$
- $y = (-x)^3$



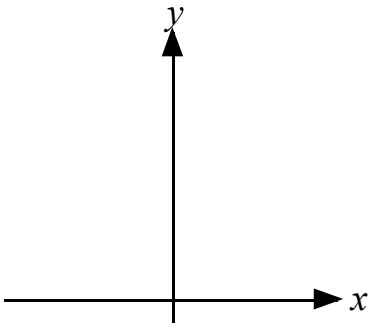
- $y = 2x$
- $y = 2(-x)$



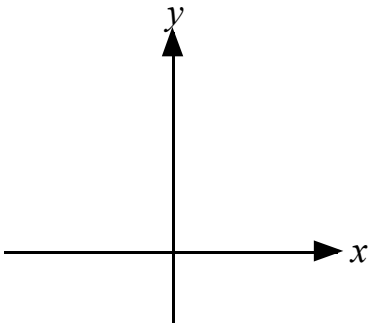
- $y = x^2 + 2$
- $y = (-x)^2 + 2$



- $y = x^2 - 4x$
- $y = (-x)^2 - 4(-x)$



- $y = 2^x$
- $y = 2^{-x}$



- $y = x^3 + 3x^2 + 3$
- $y = (-x)^3 + 3(-x)^2 + 3$

Complete this statement:

General conclusion

Given the graph of $y = f(x)$ to draw the graph of $y = f(-x)$ you_____

_____

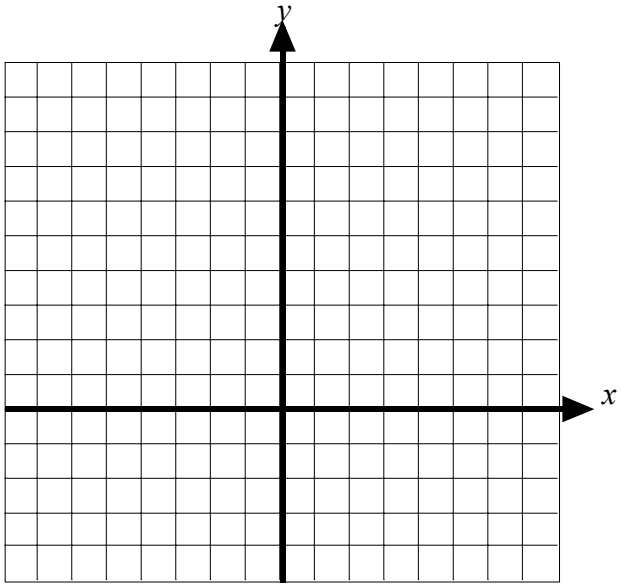
_____

Questions **DO NOT USE YOUR GRAPHIC CALCULATOR**

1. (a) On the blank grid, sketch the graph of $y = x^2$

   (b) On the same grid sketch the function $y = x^2 + 2$.

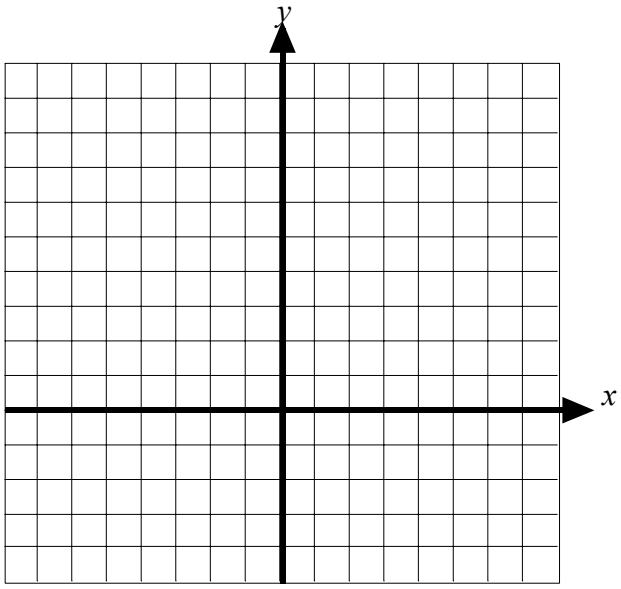
   (c) On the same grid sketch the function $y = x^2 - 3$.



2. (a) On the blank grid, sketch the graph of $y = x^2$

   (b) On the same grid sketch the function $y = (x - 1)^2$

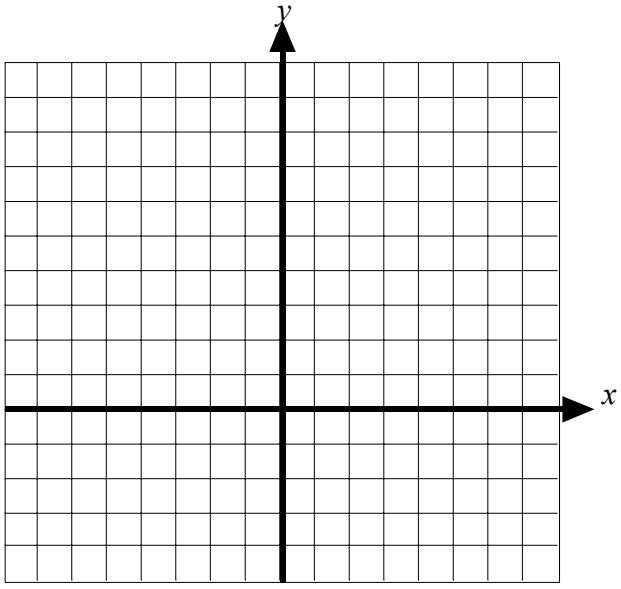
   (c) On the same grid sketch the function $y = (x + 3)^2$



3. (a) On the blank grid, sketch the graph of $y = x^2$

   (b) On the same grid sketch the function $y = (-x)^2$

   (c) On the same grid sketch the function $y = (-x)^2 + 3$



4. The graph of a function $f(x)$ is shown .

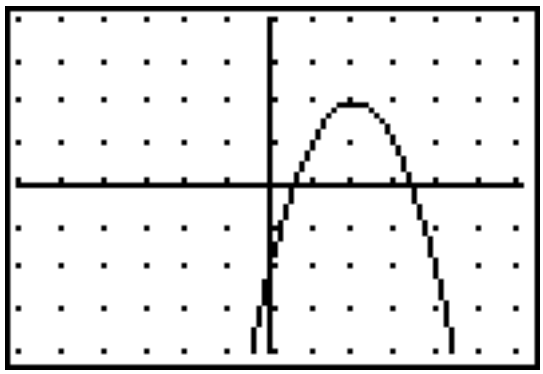
On the grids below sketch the following functions.

(a) $f(x)+1$.

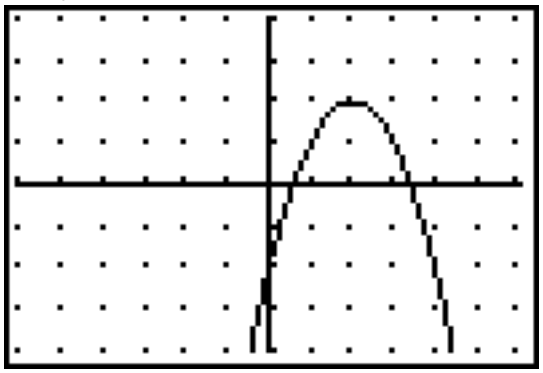
(b) $f(x)+2$.

(c) $f(x+2)$.

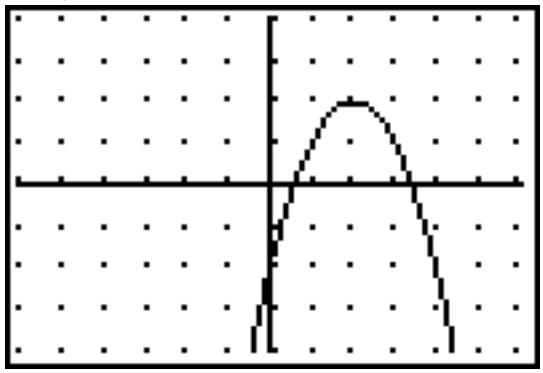
(d) $-f(x)$.



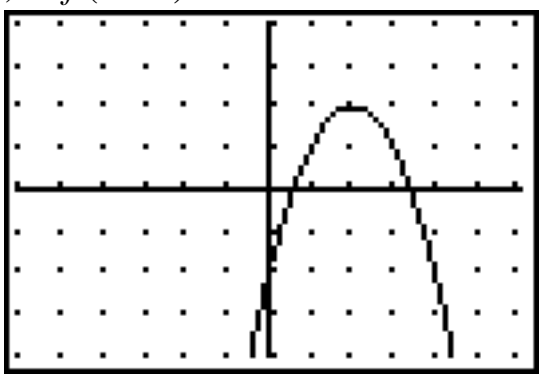
(a) $f(x)+1$.



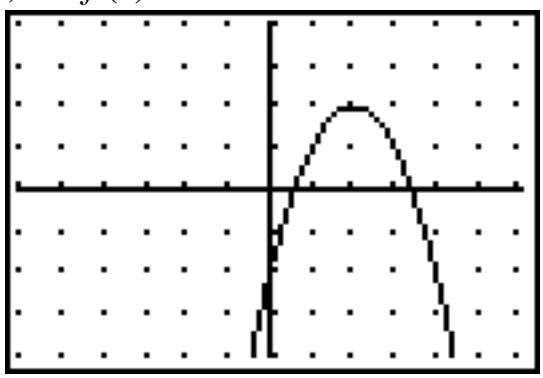
(b) $f(x)+2$.



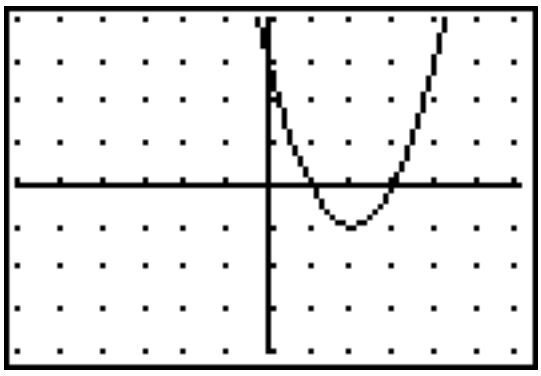
(c) $f(x+2)$



(d) $-f(x)$.



5. The graph of a function $g(x)$ is shown.



Sketch the following functions:

(a) $g(x) + 2$.

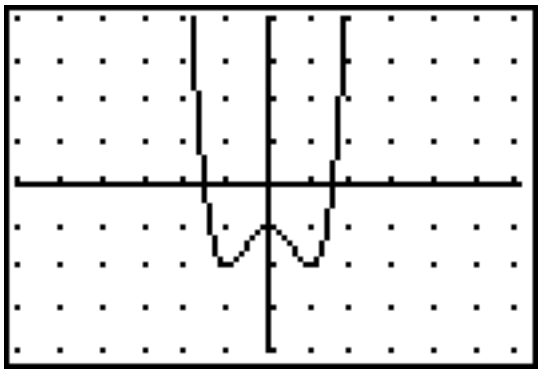
(b) $g(x) - 2$.

(c) $g(x + 2)$.

(d) $g(x + 2) + 2$.

(e) $-g(x)$.

6. The graph of a function $h(x)$ is shown.



Sketch the following functions:

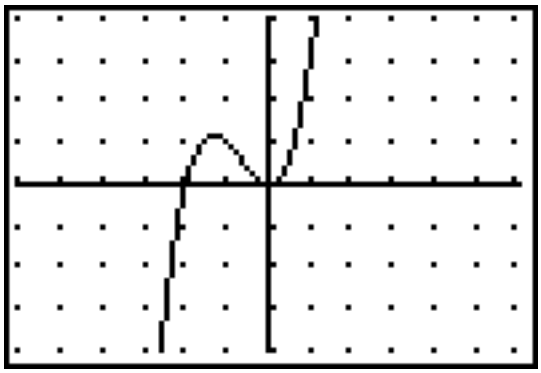
(a) $h(x) - 1$.

(b) $h(x - 1)$.

(c) $h(x + 2)$.

(d) $h(x + 2) + 1$.

7. The graph of a function $g(t)$ is shown.



Sketch the following functions:

(a) $g(t) - 4$.

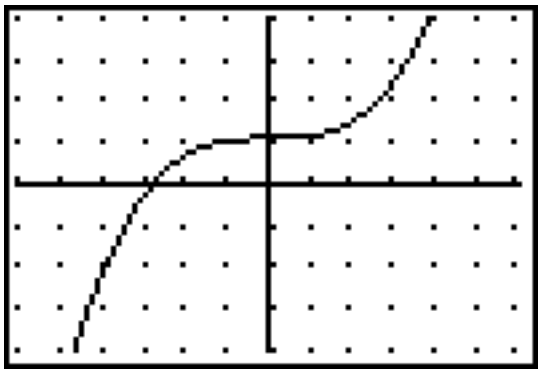
(b) $g(t + 2)$.

(c) $-g(t)$.

(d) $-g(t) + 3$.

(e) $g(-t)$.

8. The graph of a function $f(x)$ is shown.



Sketch the following functions:

(a) $f(x) + 4$.

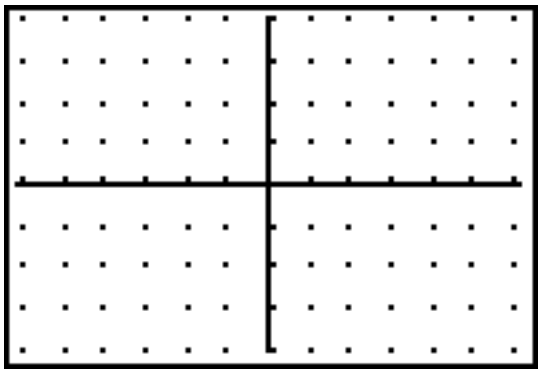
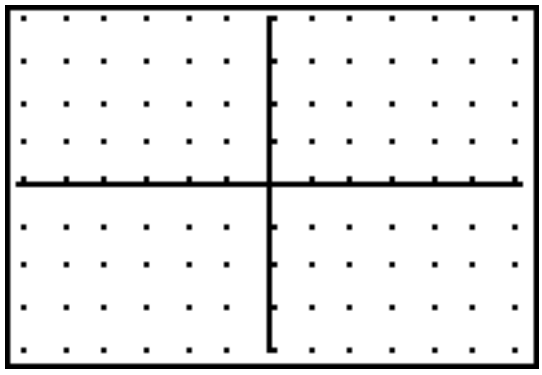
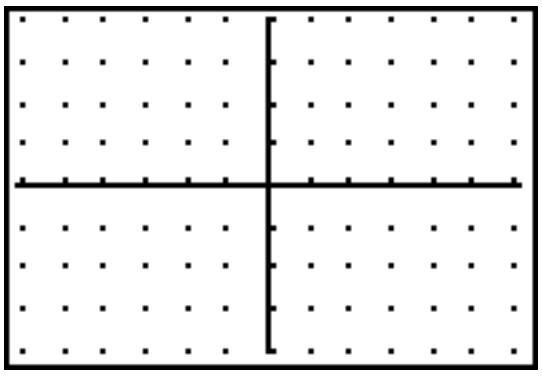
(b) $f(x + 2)$.

(c) $-f(x)$.

(d) $f(-x)$.

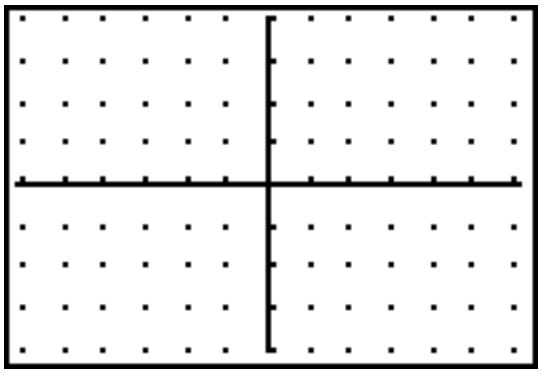
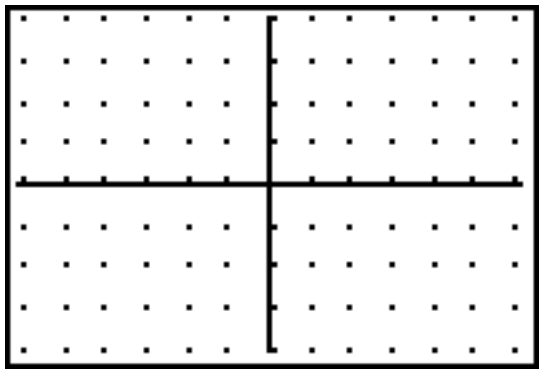
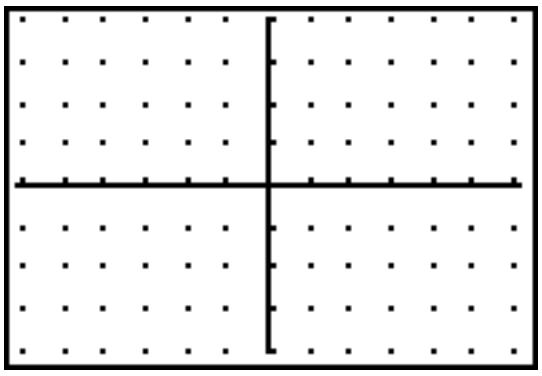
Grids for Questions 5 - 8 and for students own functions




Grids for Questions 5 - 8 and for students own functions



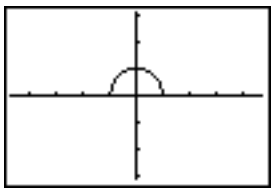

Functions and Transformations Challenge

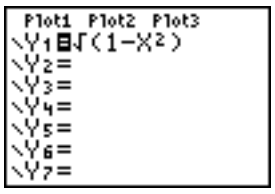
By using your knowledge of transformations.

Can you create the the screen dumps below starting with $Y1 = \sqrt{(1 - x^2)}$.
Use ZDecimal window.

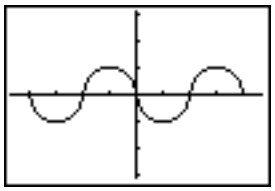
You must define each tranformation in terms of Y1.
e.g. Y2 = Y1(X - 2) + 2

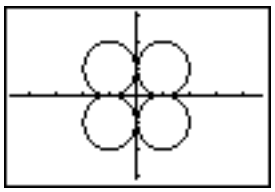
Start with this.

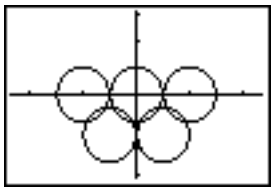




Create these.





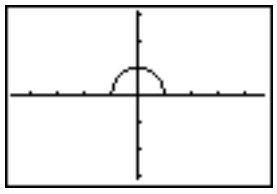


Are there alternative solutions?

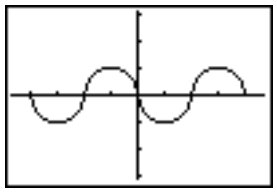
If so can you find them?

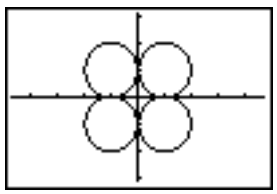
Functions and Transformations Challenge
A Solution

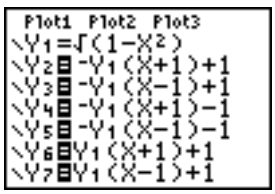
Start with this.

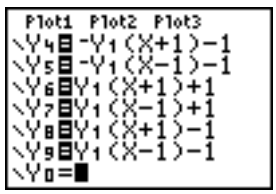


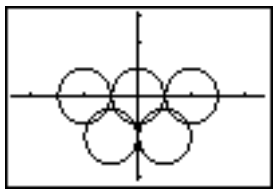
Create these.

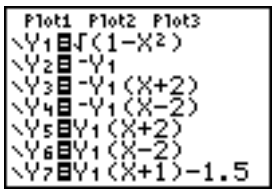


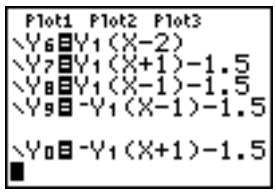












Other solutions can be found by defining subsequent transformations in terms of previous ones. i.e. not all in terms of Y1.

It is also possible to draw some of them as circles using the DRAW menu ! !